Visibility Solutions for Sensor Networks and UAVs

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PhD Defense

Acknowledgments

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Visibility Problem Prototype

How should an autonomous system move such that it achieves or maintains line of sight of some object(s) of interest in a nonconvex environment?

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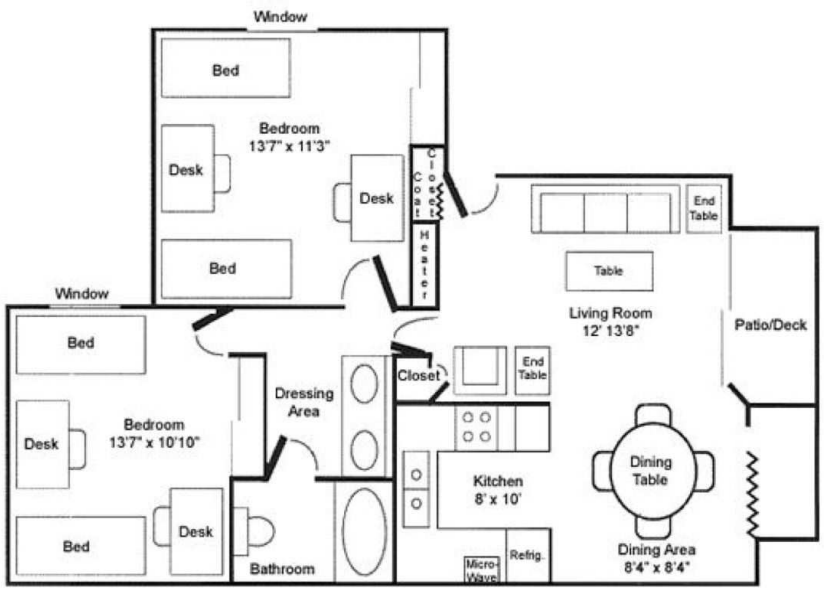

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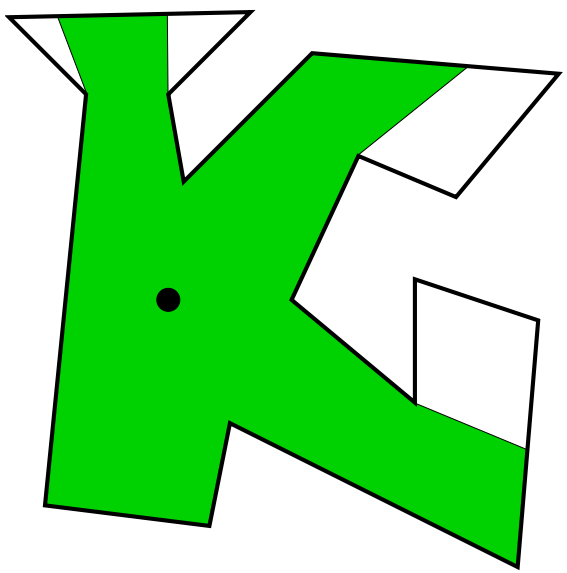
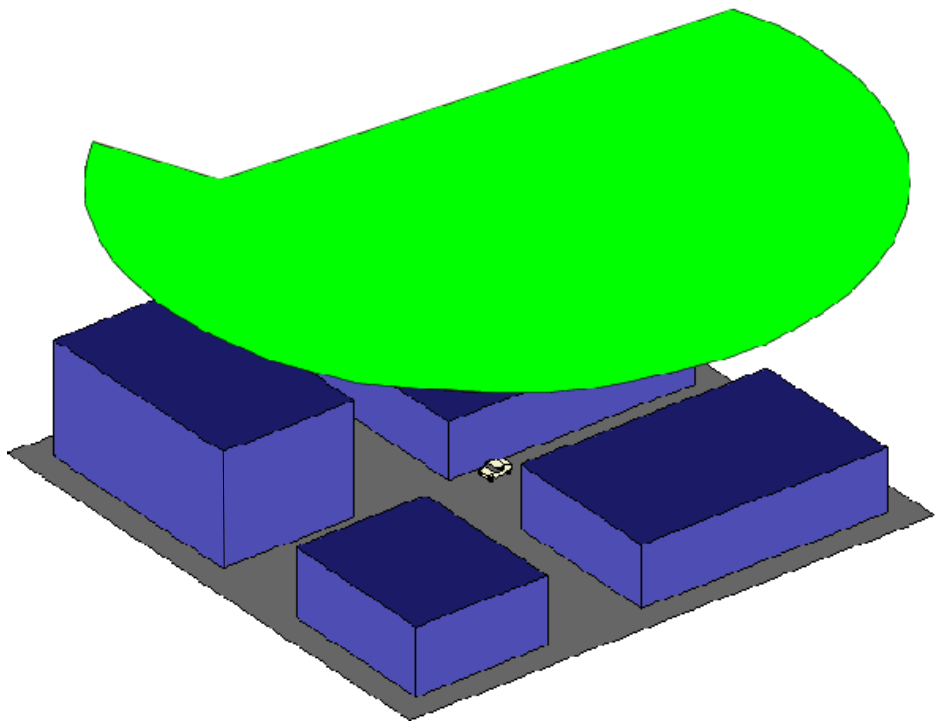




Visibility-Based Pursuit-Evasion

Visual Surveillance

Visual Reconnaissance

Exploration

Outline

Visibility-Based Pursuit-Evasion
Part I: Searchlight and Camera Scheduling

Visual Surveillance
Part II: Multi-Agent Deployment

Visual Reconnaissance
Part III: UAV Path Planning

Selected Publications

Journal Articles and Conference Proceedings

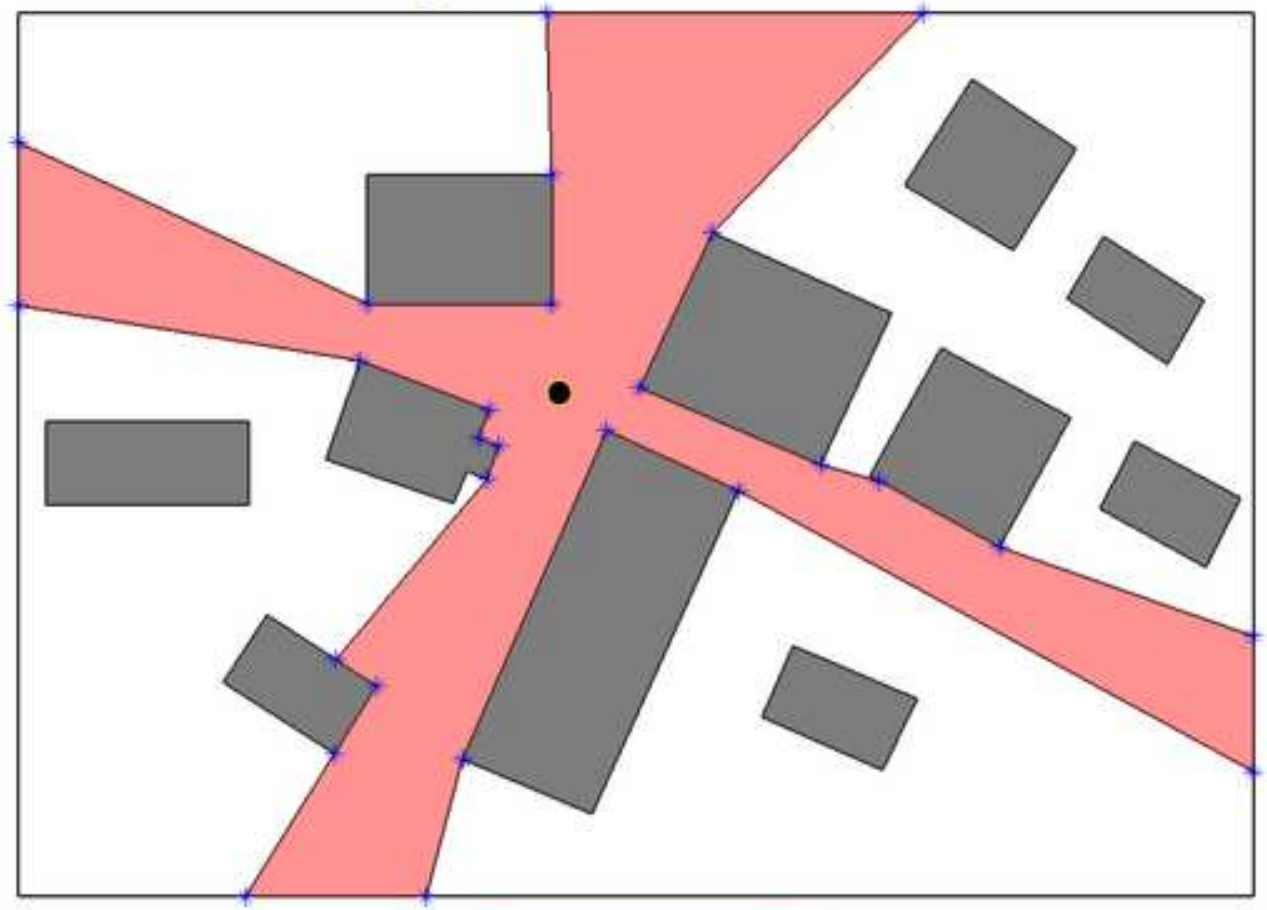
A Complete Algorithm for Searchlight Scheduling – Obermeyer, Ganguli, and Bullo, in *International J. of Comp. Geometry and Applications*, Note: Submitted.

Asynchronous Distributed Searchlight Scheduling – Obermeyer, Ganguli, and Bullo, in *IEEE Conf. on Decision and Control*, 2007.

Multi-Agent Deployment for Visibility Coverage in Polygonal Environments with Holes – Obermeyer, Ganguli, and Bullo, Note: Journal Article in Preparation.

Sampling-Based Roadmap Methods for a Visual Reconnaissance UAV – Obermeyer, Oberlin, and Darbha, *AIAA J. of Guidance, Control, and Dynamics*, Note: Submitted.

VisiLibity.org for Floating-Point Visibility Computations



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Math behind VisiLibity
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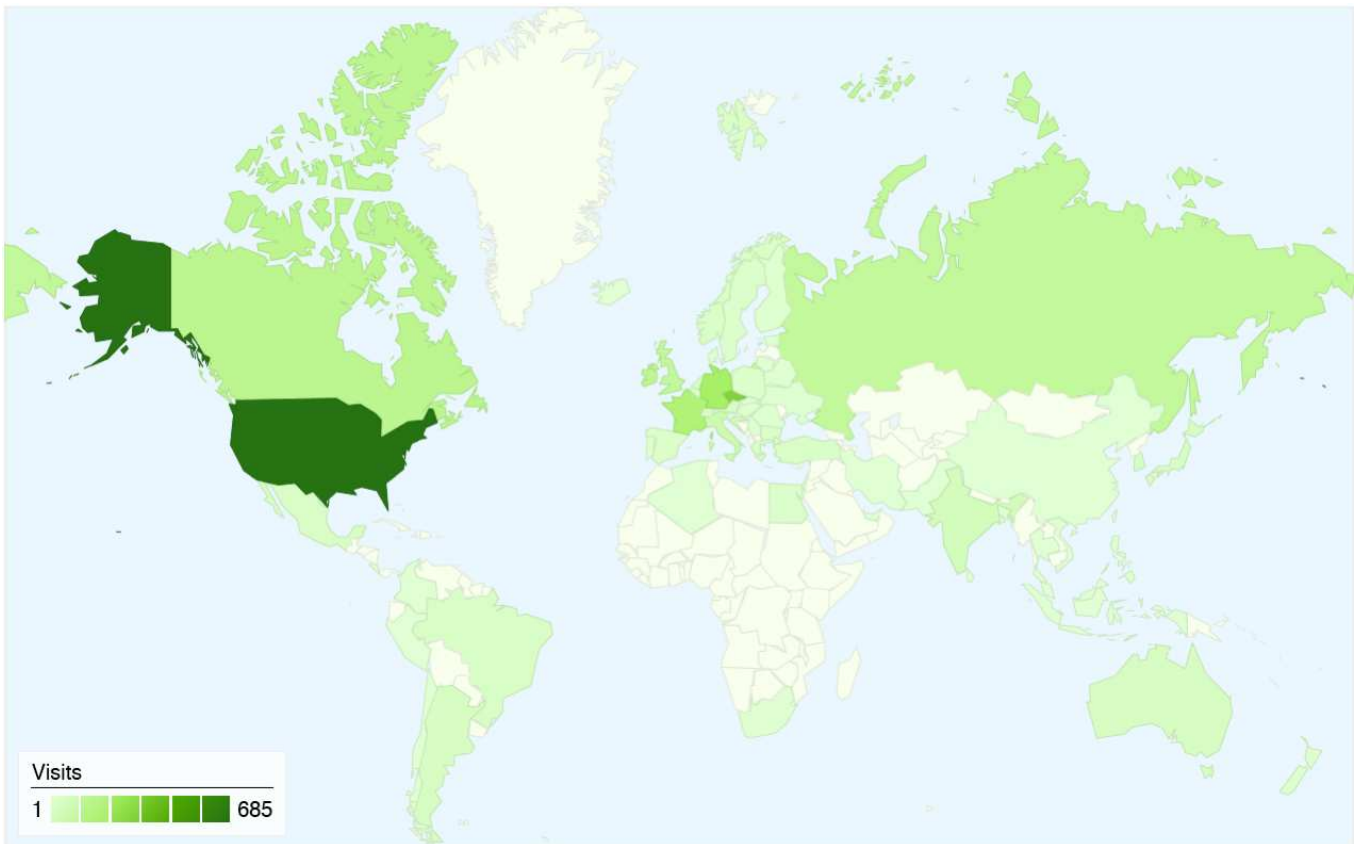
VisiLibity Stats (May 2008 – May 2010)

- 1500 unique visitors in 66 countries
- 300 downloads



Part I

Searchlight and Camera Scheduling

The Searchlight Scheduling Problem

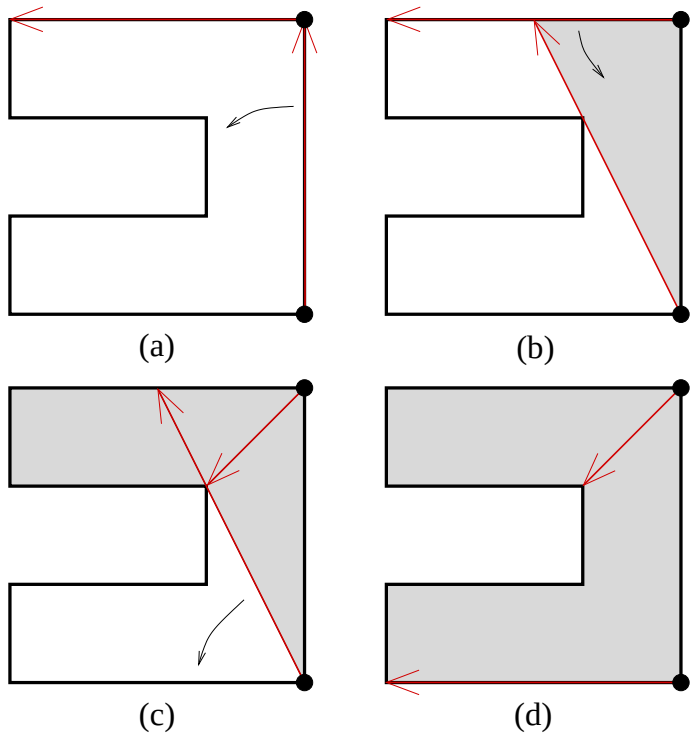
[Sugihara, Suzuki, and Yamashita, 1990]

Goal

Find a **schedule** *to rotate a set of searchlights such that any evader in an environment will necessarily be detected in finite time*

Assumptions

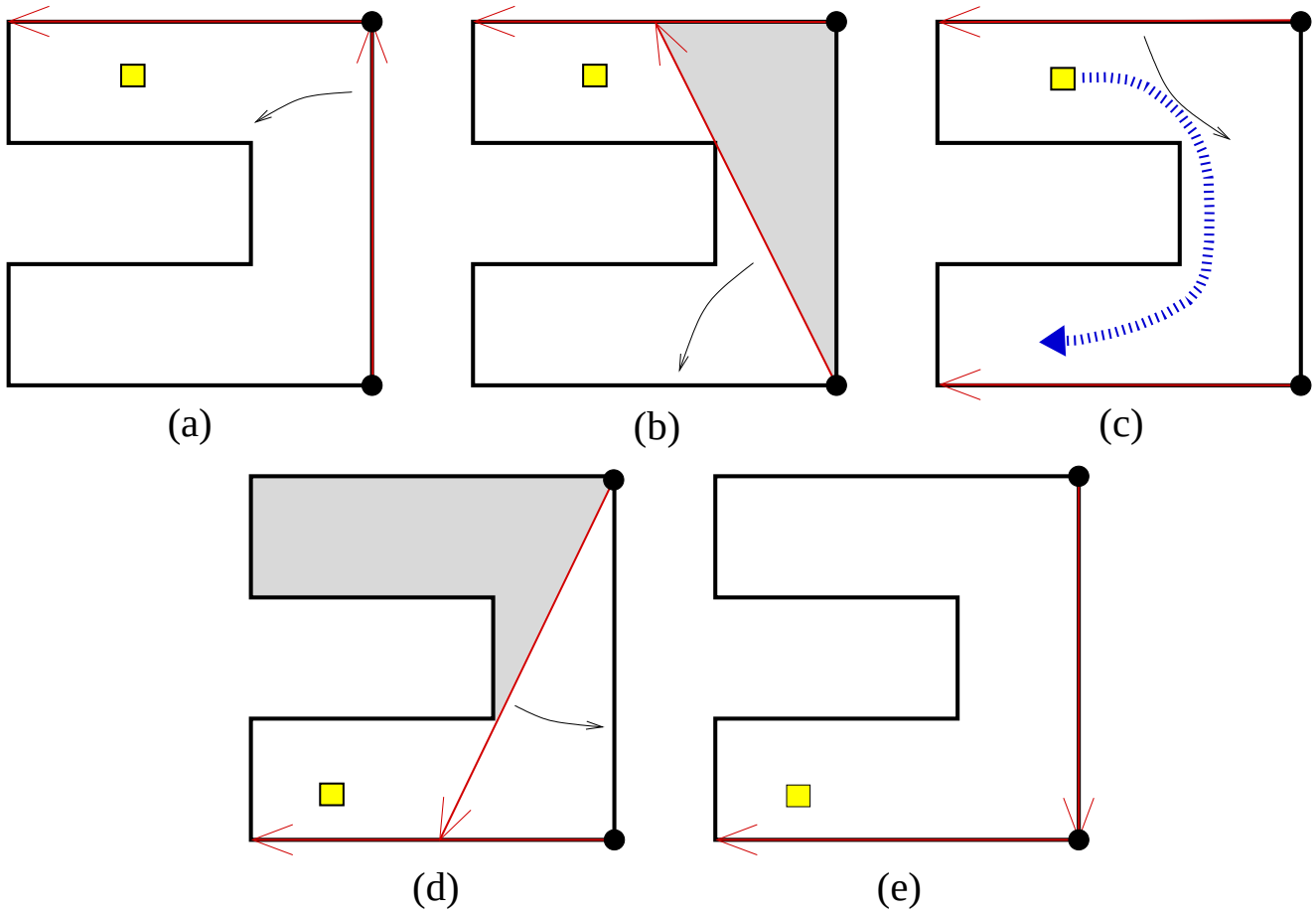
- Polygonal environment
- No colocated searchlights
- Evaders have unbounded speed



The Searchlight Scheduling Problem

[Sugihara, Suzuki, and Yamashita, 1990]

Example of a schedule which fails



The Searchlight Scheduling Problem

[Sugihara, Suzuki, and Yamashita, 1990]

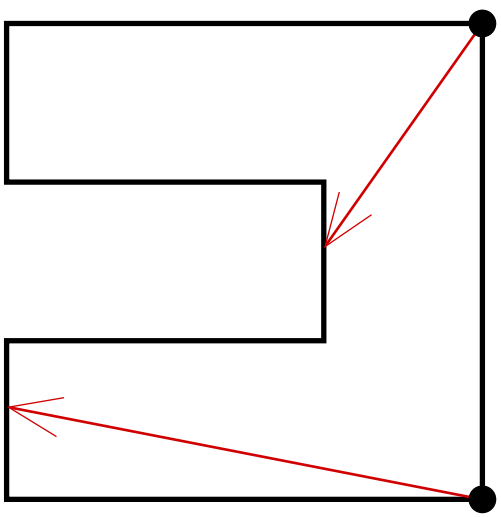
Schedule for a single searchlight

$\theta^{[i]} : [0, T] \longrightarrow \mathbb{T}^1$

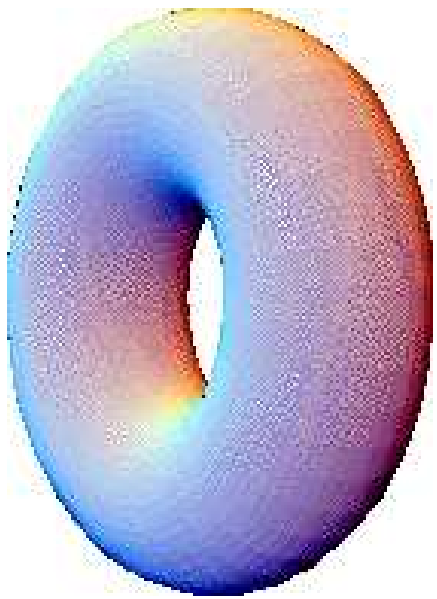
Joint Schedule for N searchlights

$\Theta^{[i]} : [0, T] \longrightarrow \mathbb{T}^N$

problem instance

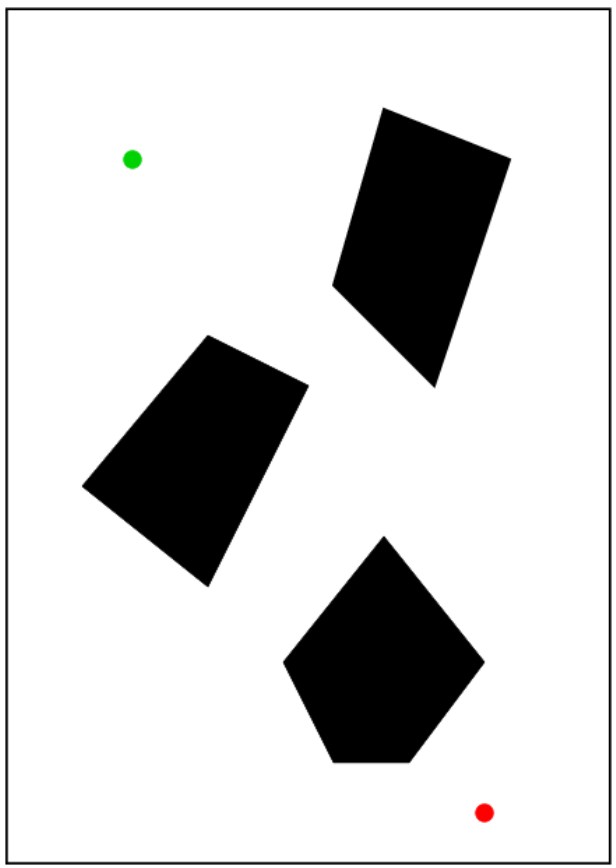


configuration space $\mathbb{T}^N$

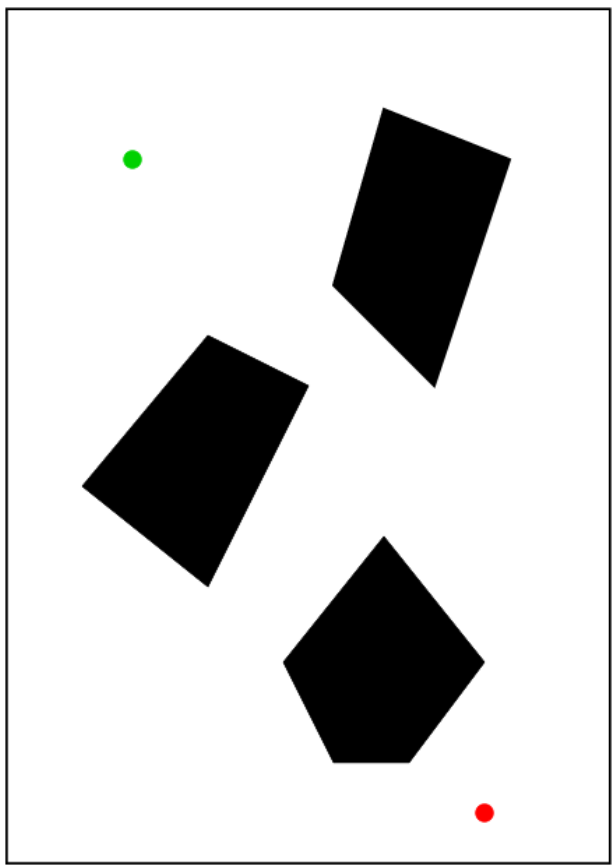


Technical Approach

Inspiration from Euclidean Shortest Path Problem



Reduction Theorem [Nilsson, 1969]
Shortest paths hug corners



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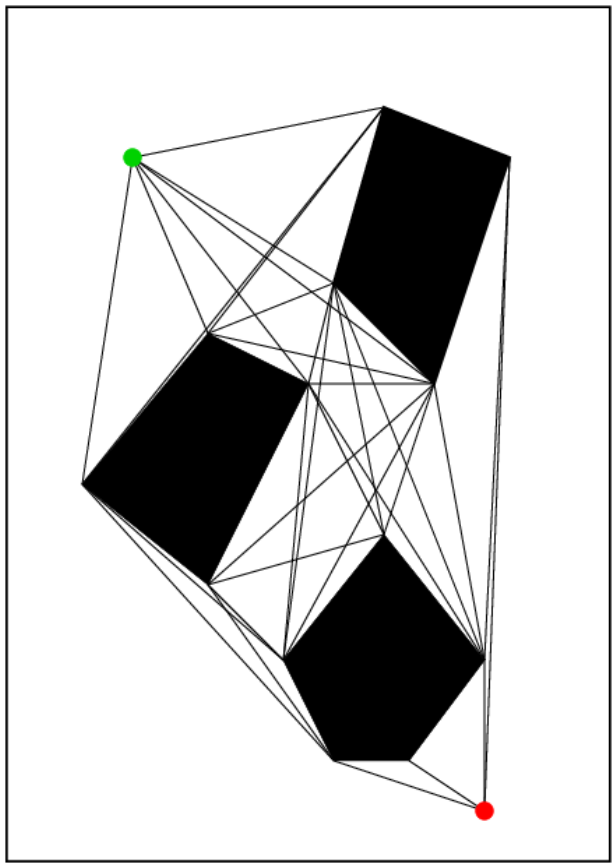

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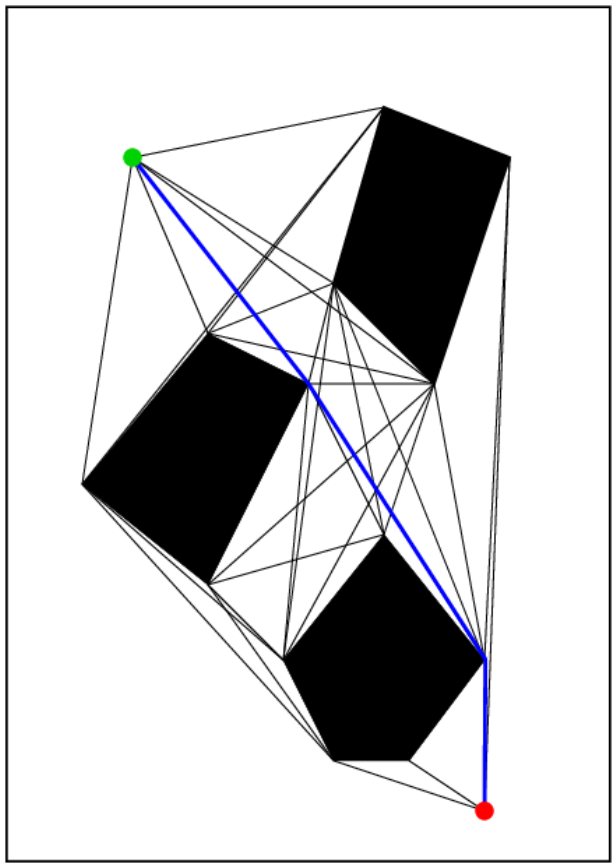

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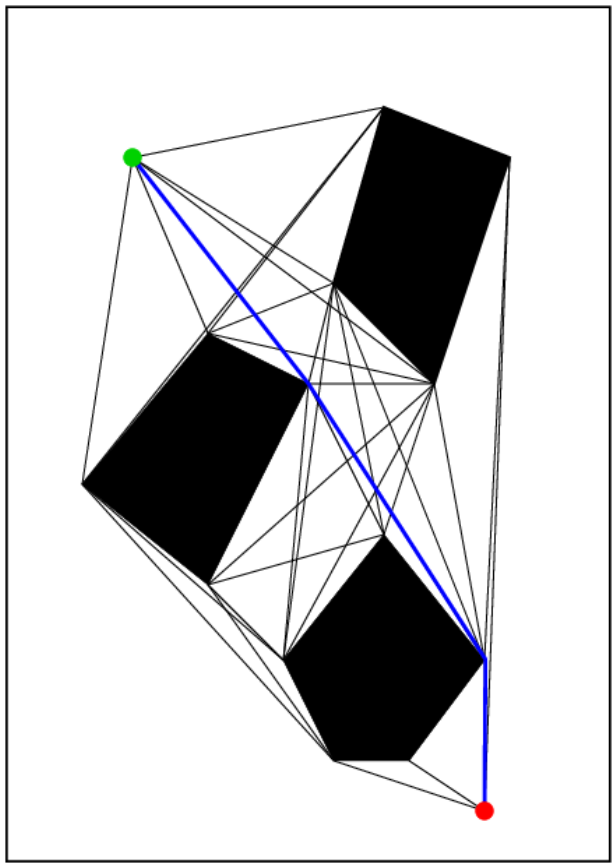
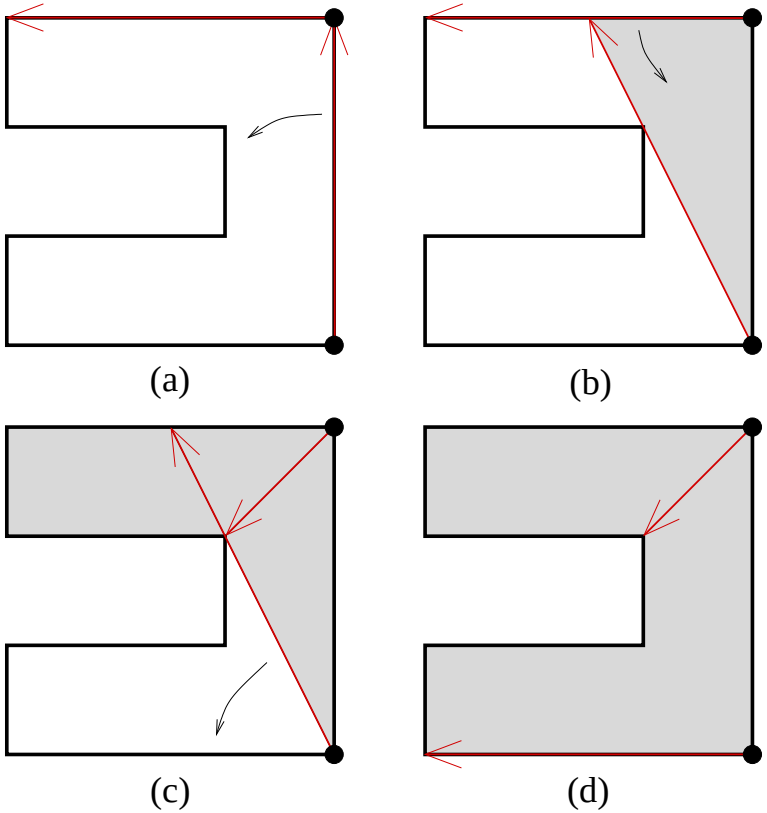


Roadmap Method*

(1) discretize continuous space, then
(2) search resulting graph with shortest path algorithm

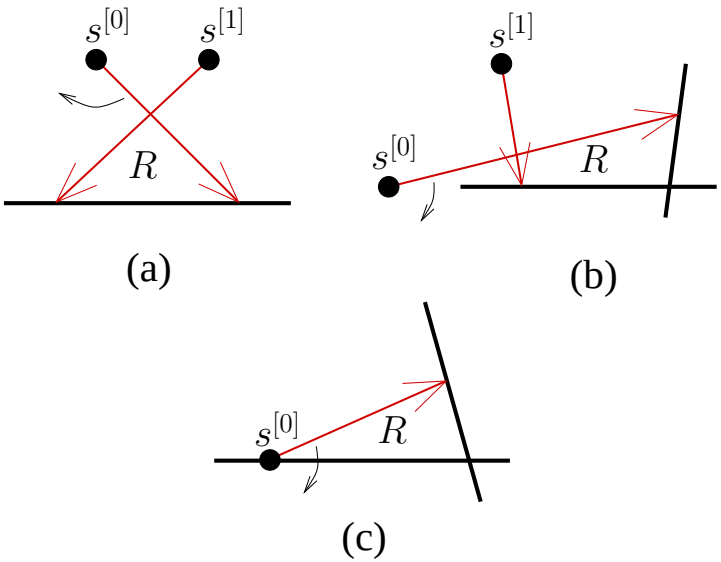
*First applied to visibility-based pursuit-evasion in [Guibas et al, 1999]

Definition: Maximal Nonseparable Region

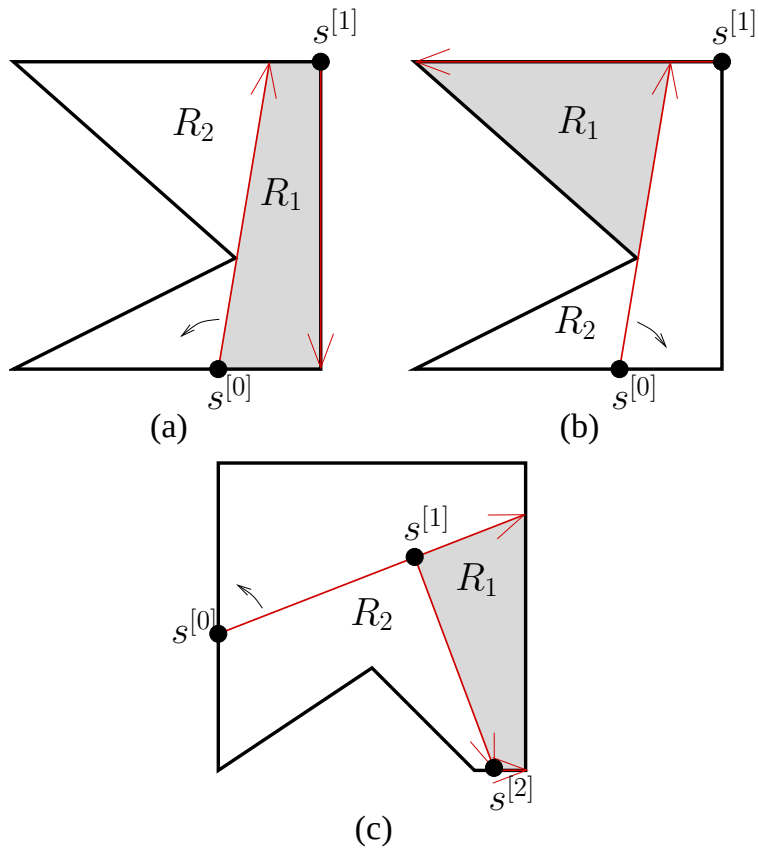


Maximal nonseparable regions undergo discrete **topological changes**:

disappear/appear



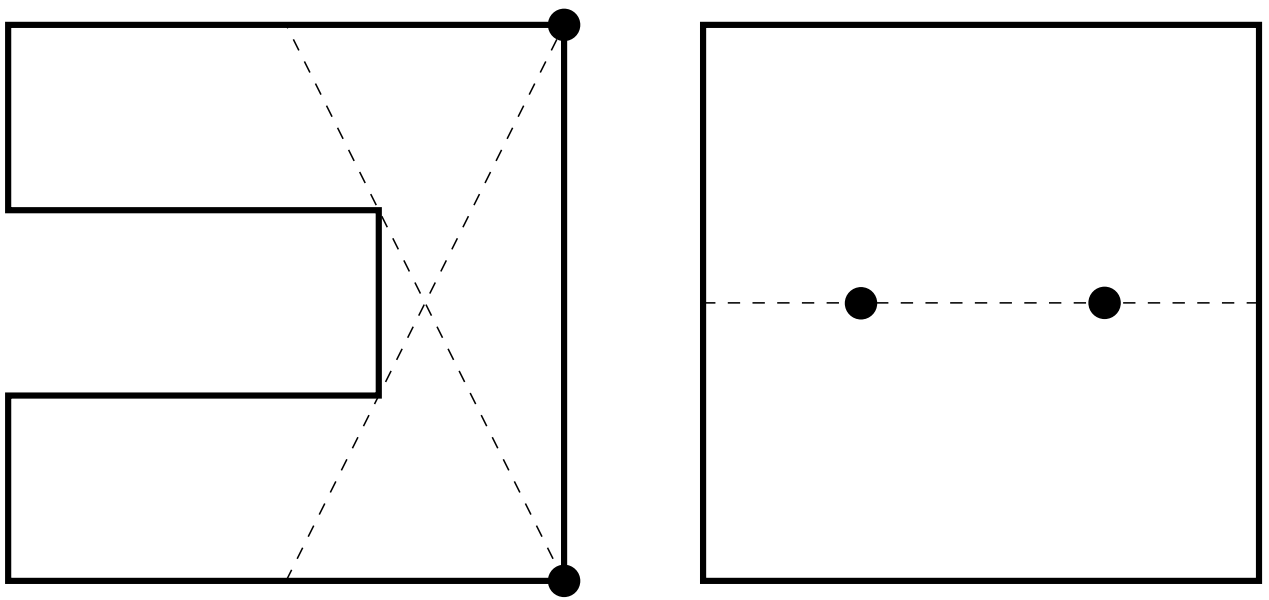
merge/split



Definition: Critical Angles

A searchlight has a **critical angle** wherever

1. it is aimed along a wall,
2. its visibility is occluded by a reflex vertex, or
3. it has line of sight to another searchlight

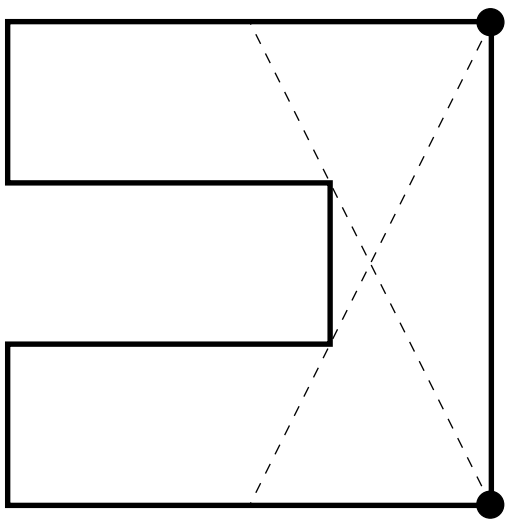


Reduction Theorem

Without loss of generality, searchlights may

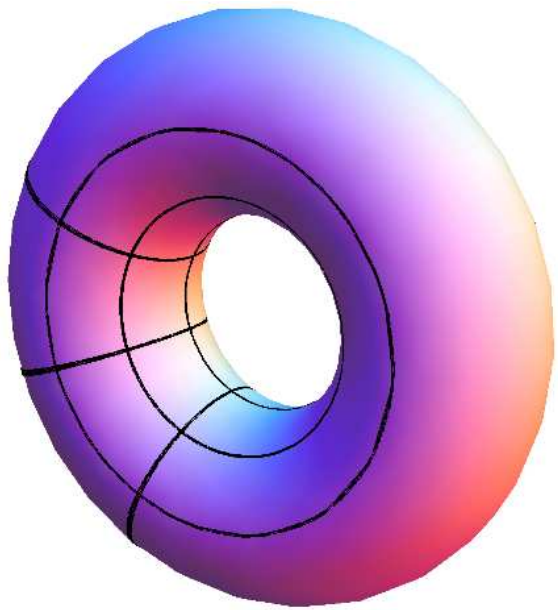
1. rotate only one at a time, and
2. only stop at critical angles

critical angles



$\Rightarrow$

roadmap in $\mathbb{T}^N$



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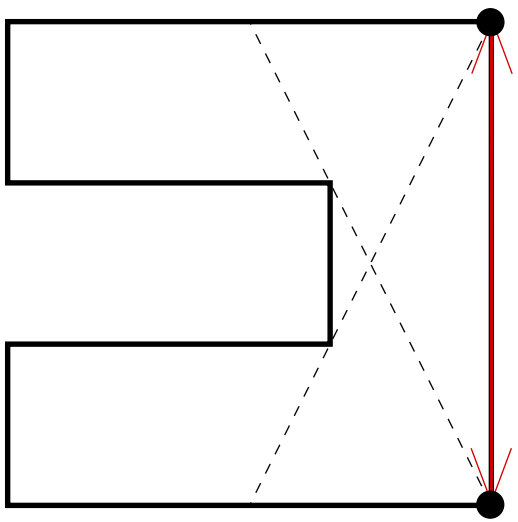

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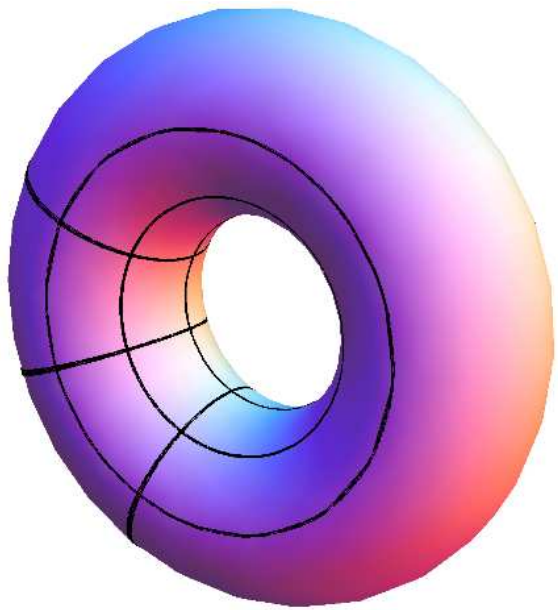

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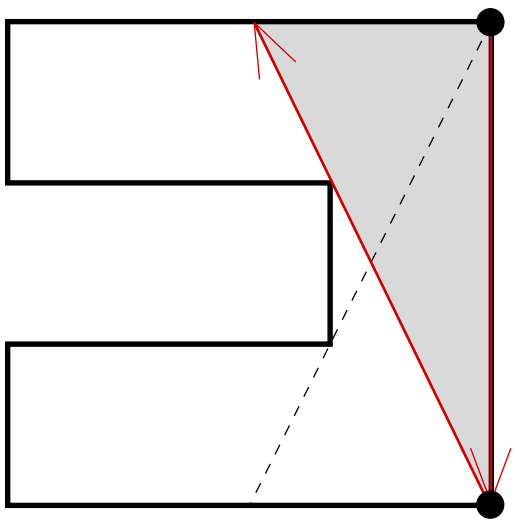

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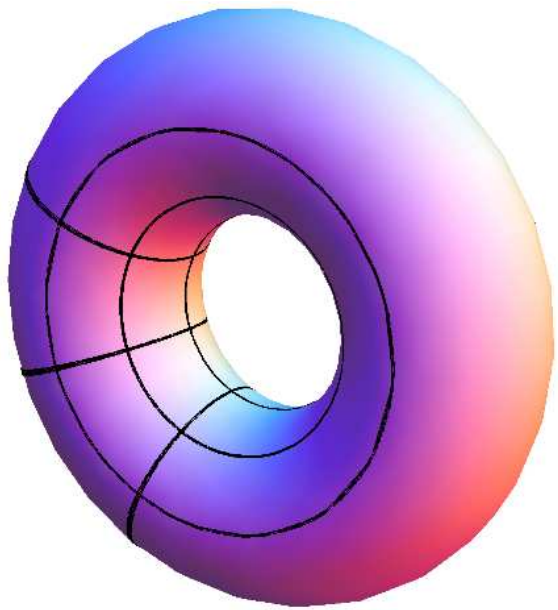

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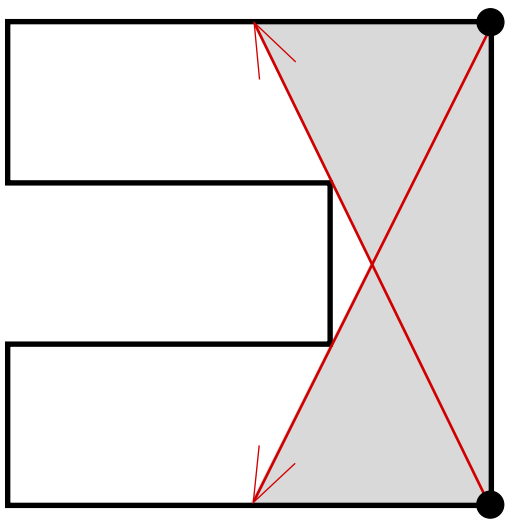

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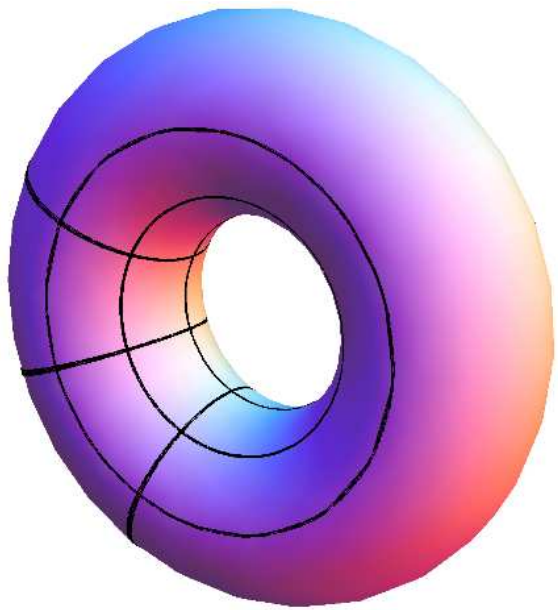

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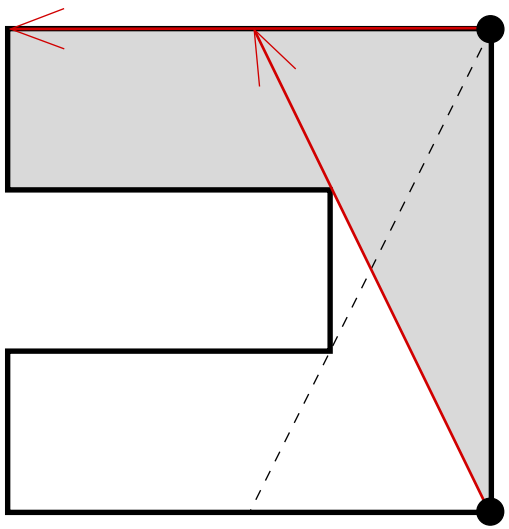

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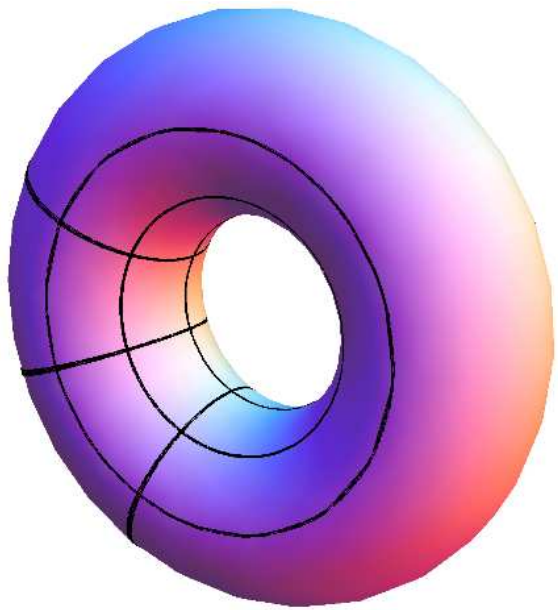

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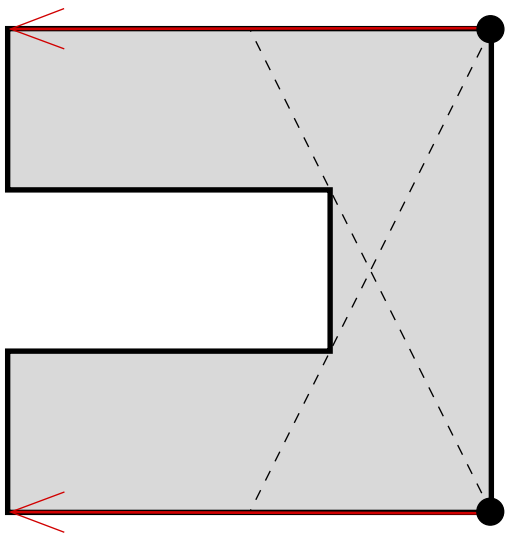

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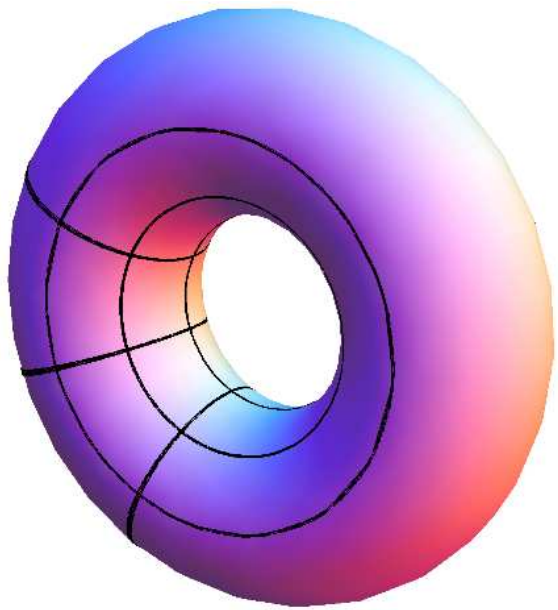


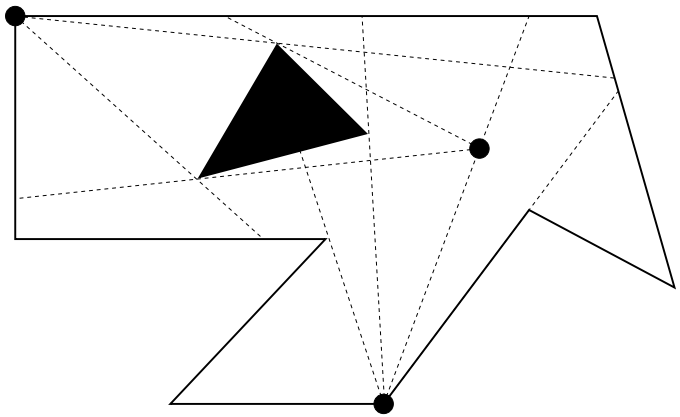
Combinatorial Roadmap Method

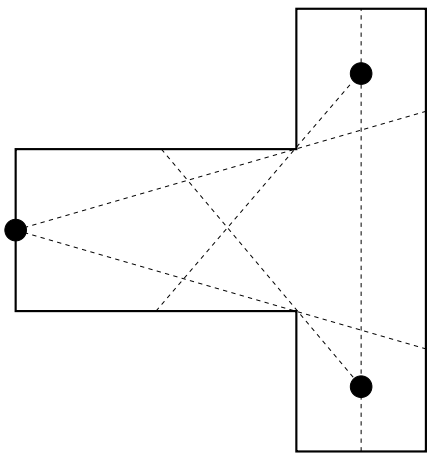
1. Find critical angles
2. Construct roadmap
3. Perform modified breadth-first search of roadmap

Computed Examples

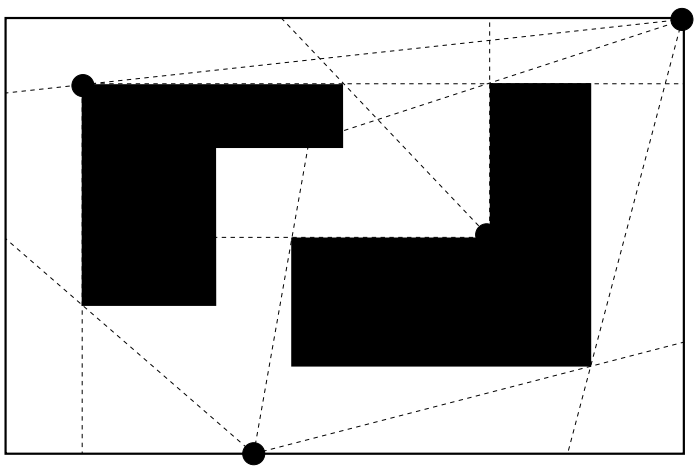
C++, 2.33 GHz i686

solutions in < 1 **second**





solution in < 4 **seconds**



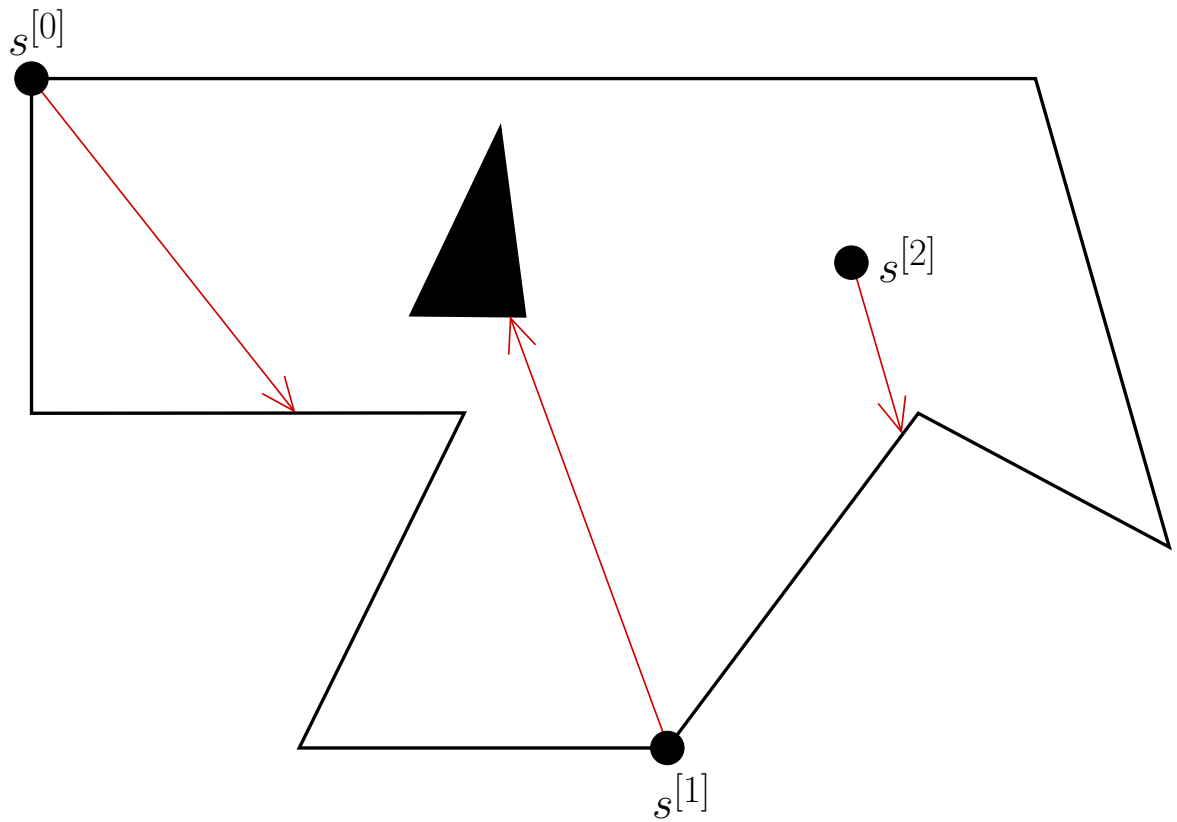
Computed Examples

C++, 2.33 GHz i686

Node Bound	**Nodes Visited**	**Computation Time**	# Actions
$\sim 10^{10}$	**1321**	$<$ **1 sec.**	12
$\sim 10^{6}$	**463**	$<$ **1 sec.**	8
$\sim 10^{9}$	**6176**	$<$ **4 sec.**	16

Goal

Given N **ϕ-searchlights** with FOVs $\phi^{[0]}, \phi^{[1]}, \phi^{[2]}, \ldots, \phi^{[N-1]}$, find a search schedule



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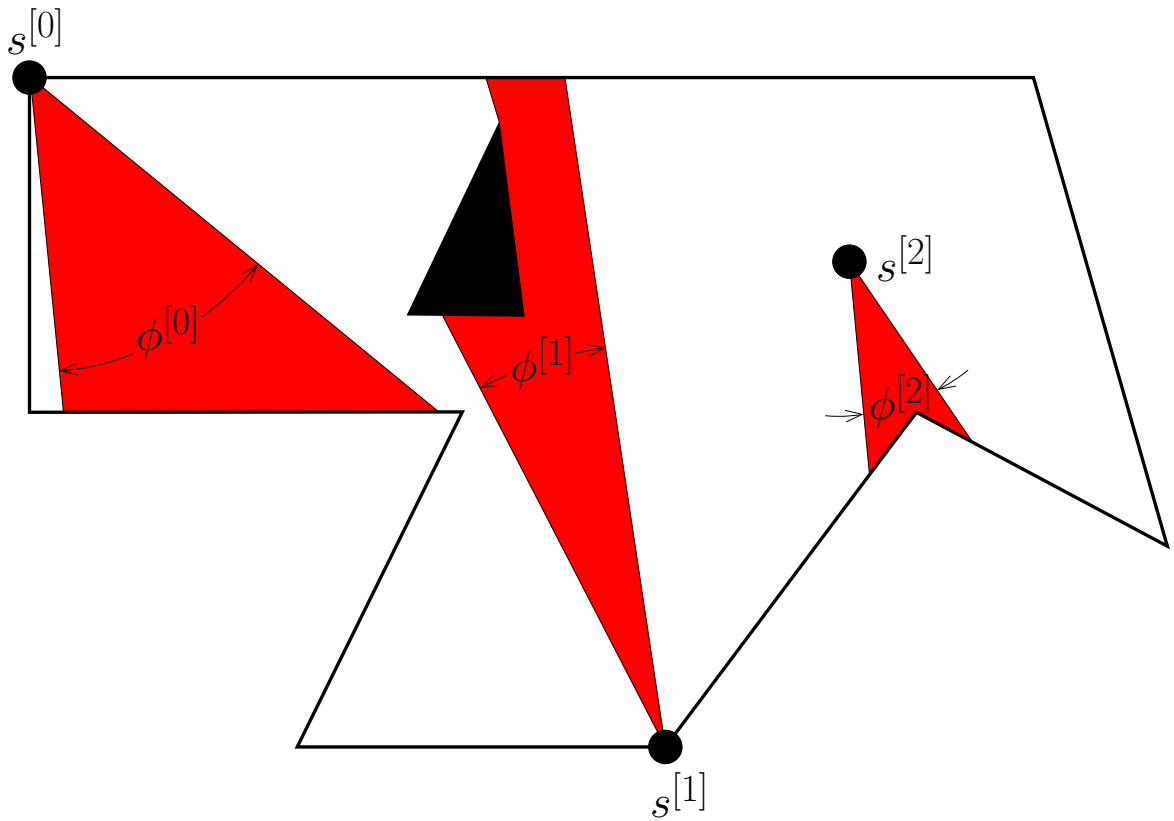
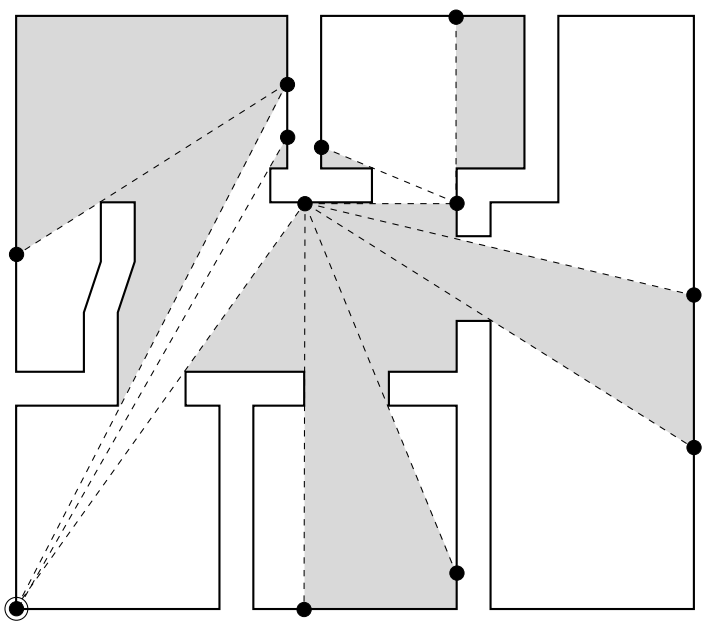


Serendipity: Searchlight Placement Scheme

Reflex Vertex Straddling (**RVS**) Positions



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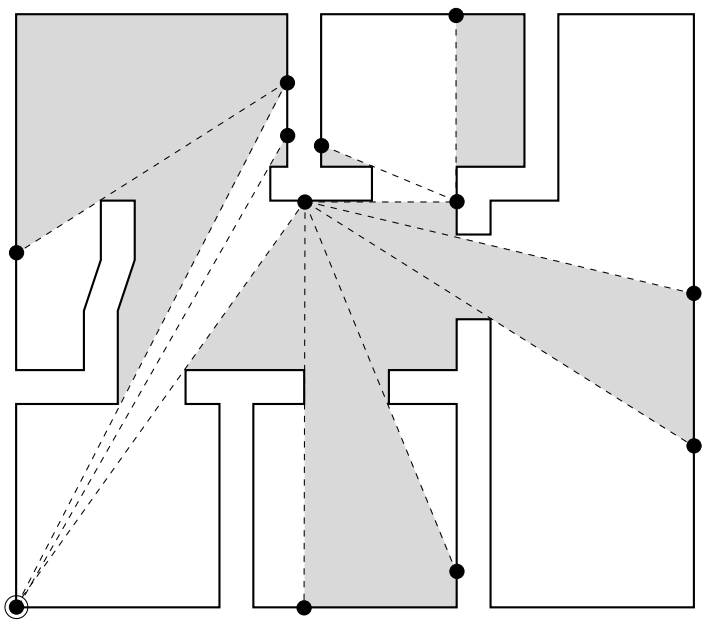


Theorem

RVS requires

- $\leq r + 1 \leq n - 2$ searchlights, or $\leq \frac{n}{2} - 2$ in ortho. environment
- time to clear $\mathcal{O}(r)$

Note: n:= total number of vertices
r:= number of reflex vertices

Part II

Multi-Agent Deployment

Distributed Deployment Problem

Goal

Design a distributed algorithm for agents to deploy into an environment such that

1. *line-of-sight connectivity is maintained, and*
2. *entire environment is visible from final positions*

Assumptions

- unmapped static polygonal environment (n vertices, h holes)
- agents identical except for UIDs
- omnidirectional vision*
- only line-of-site communication and sensing
- agents may establish only local common reference frame
- first order dynamics: $\dot{p}^{[i]} = u \leq u_{\max}$

Algorithm Design Requirements

Upper bounds on

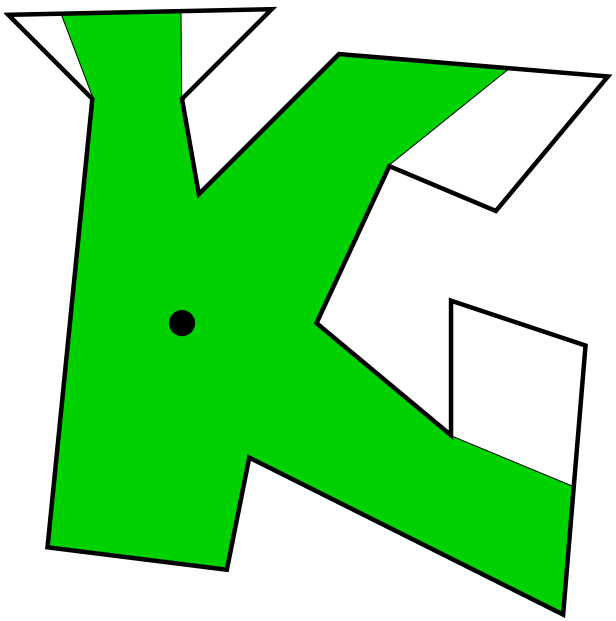
- **number of agents** required
- **time** until full coverage
- **memory** and **communication** complexity

1. Design centralized incremental partition algorithm
2. Distributedly emulate

Definitions: Visibility Polygons

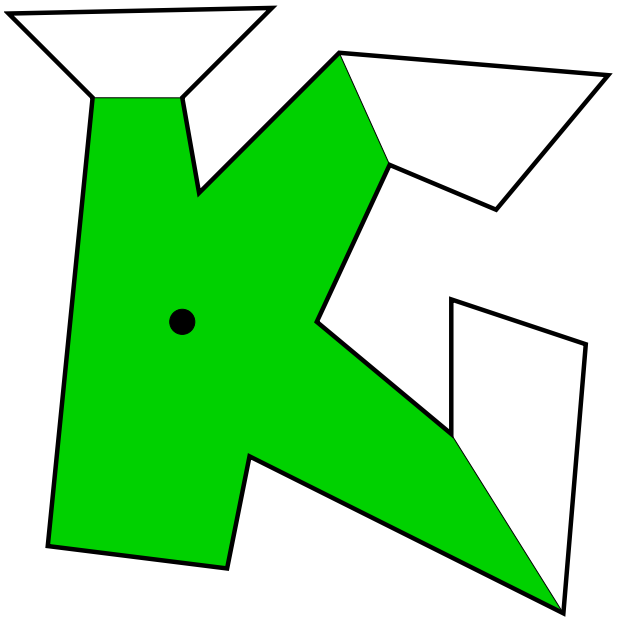
Visibility Polygon

sensing and communication



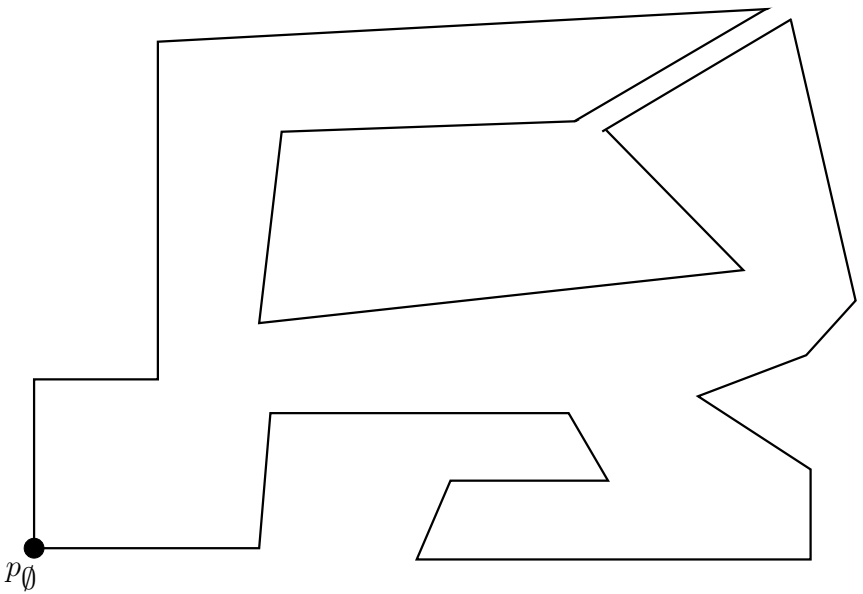
Vertex-Limited Visibility Polygon

land grabbing



Centralized Incremental Partition Algorithm

Without Holes [Ganguli et al, 2007]



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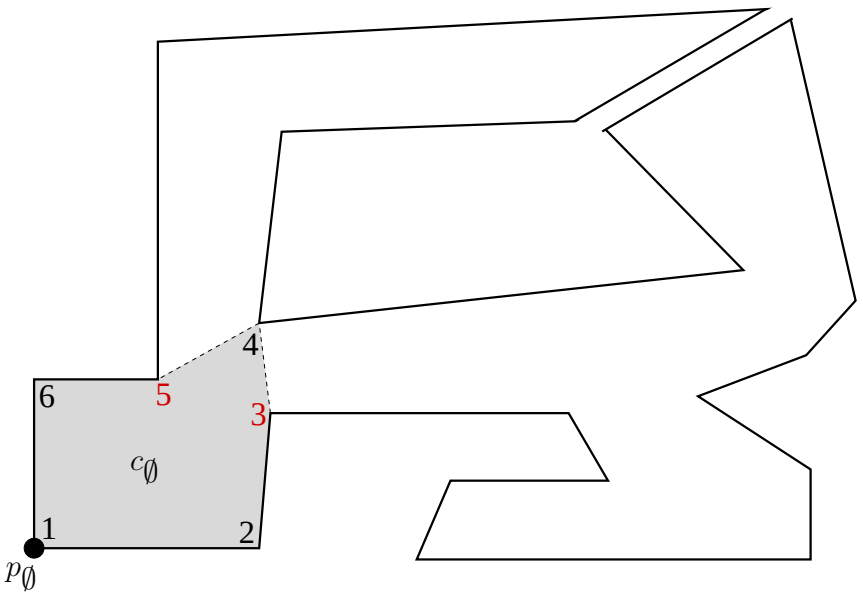

Partition Tree

$p_\emptyset, c_\emptyset$

Centralized Incremental Partition Algorithm

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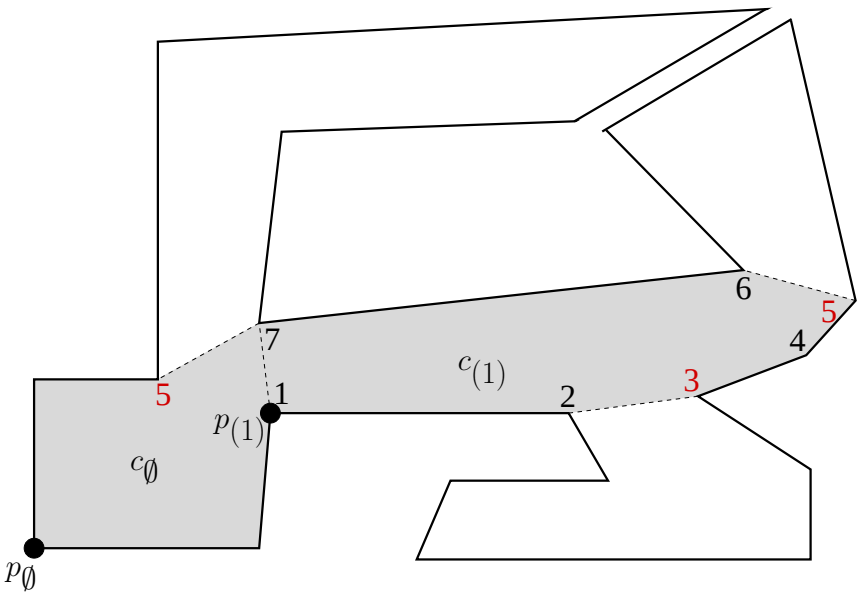

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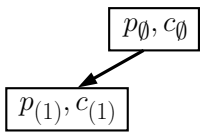

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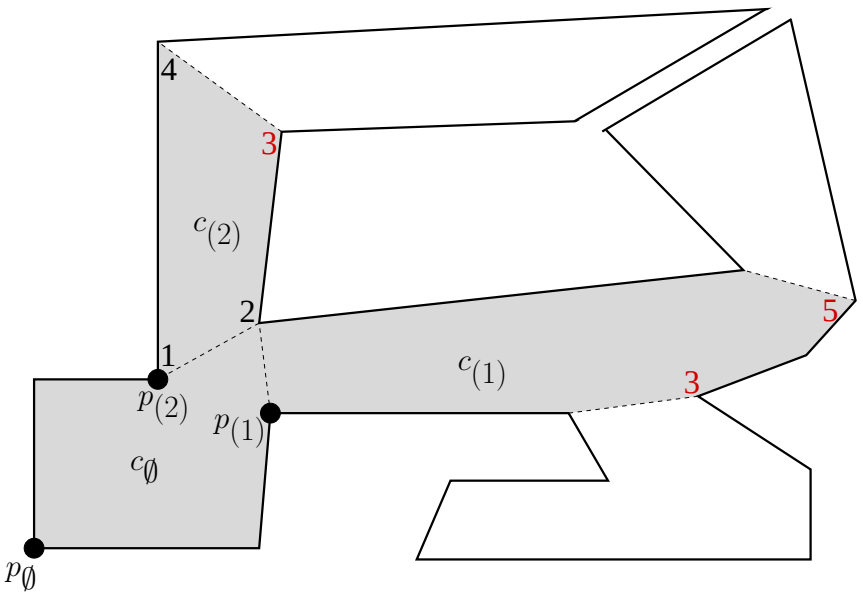

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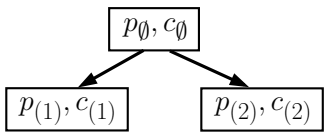

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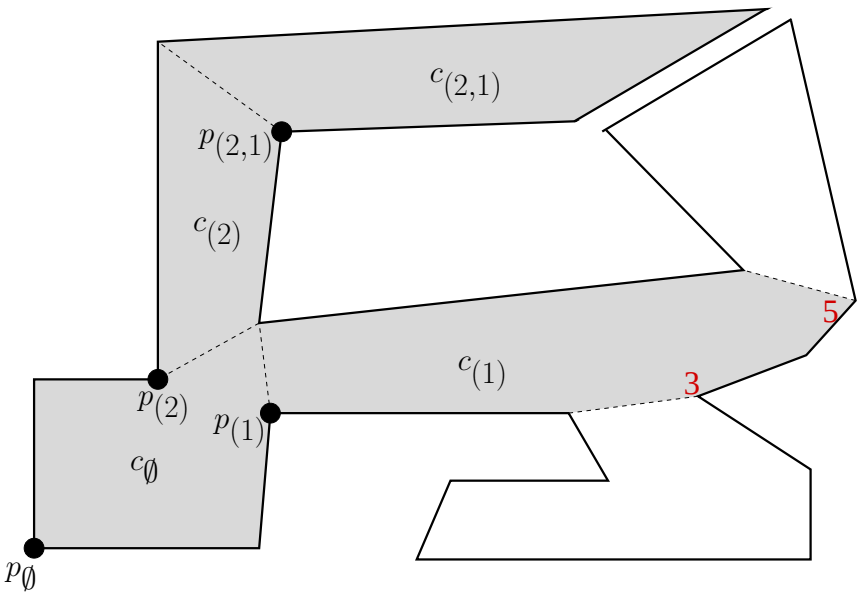
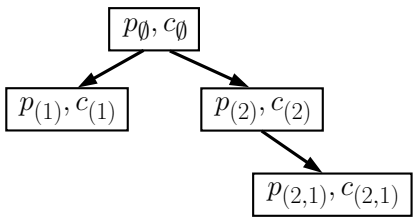
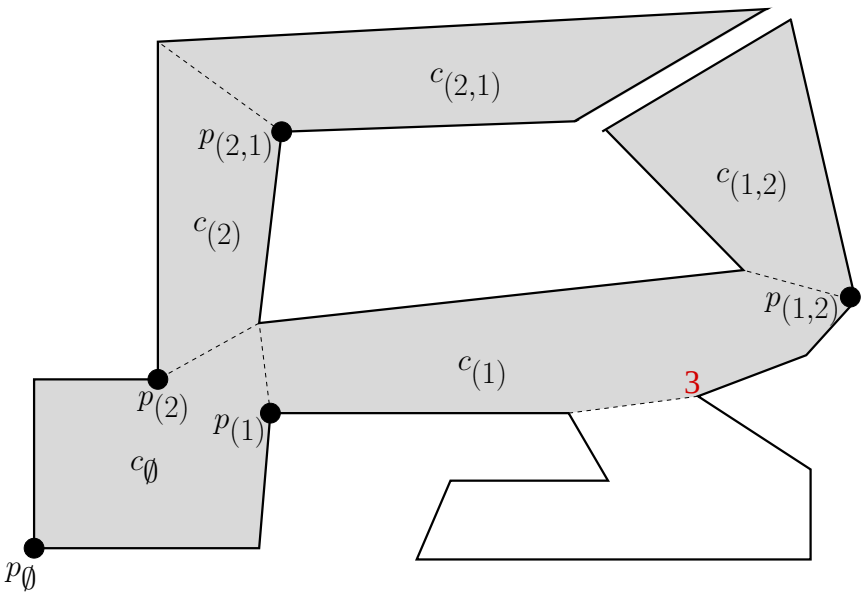


Partition Tree



Centralized Incremental Partition Algorithm

Without Holes [Ganguli et al, 2007]



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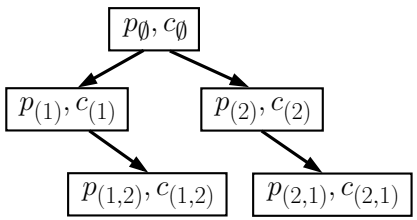

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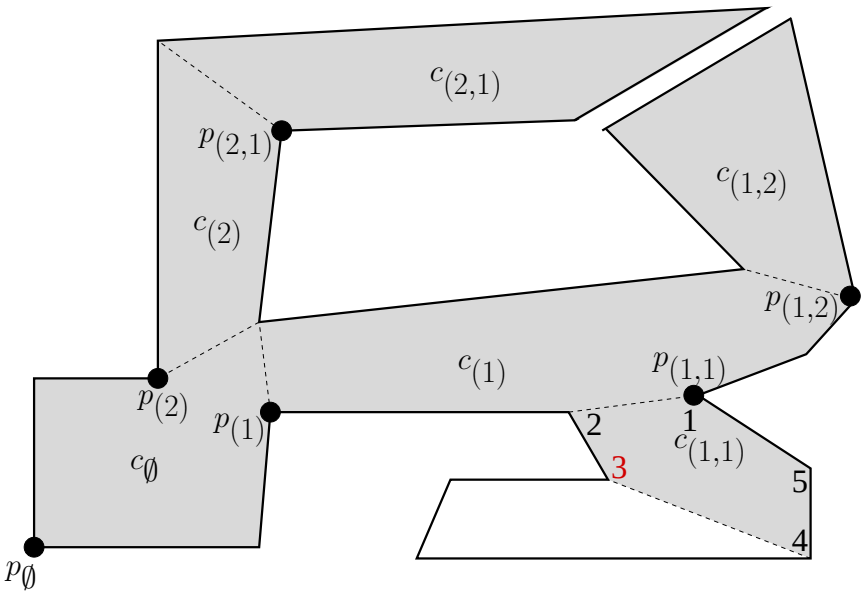

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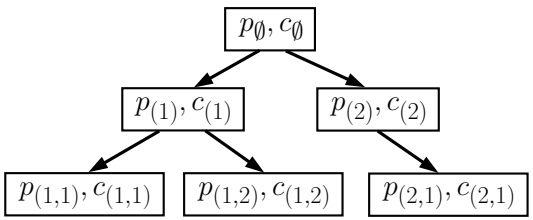

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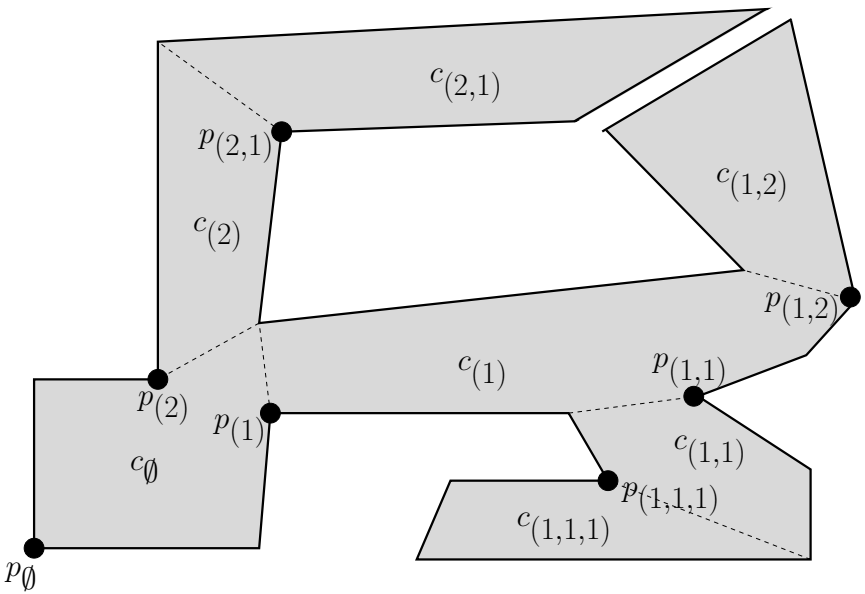

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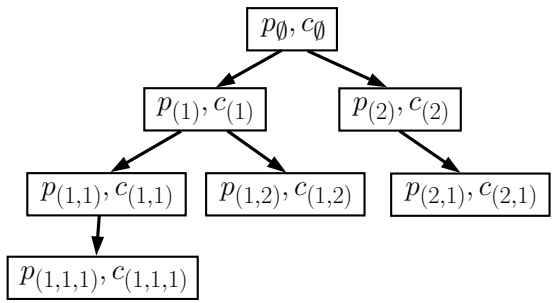

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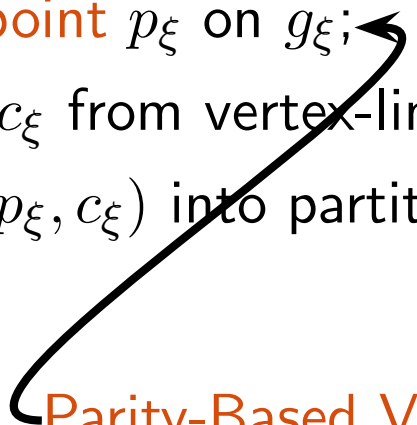
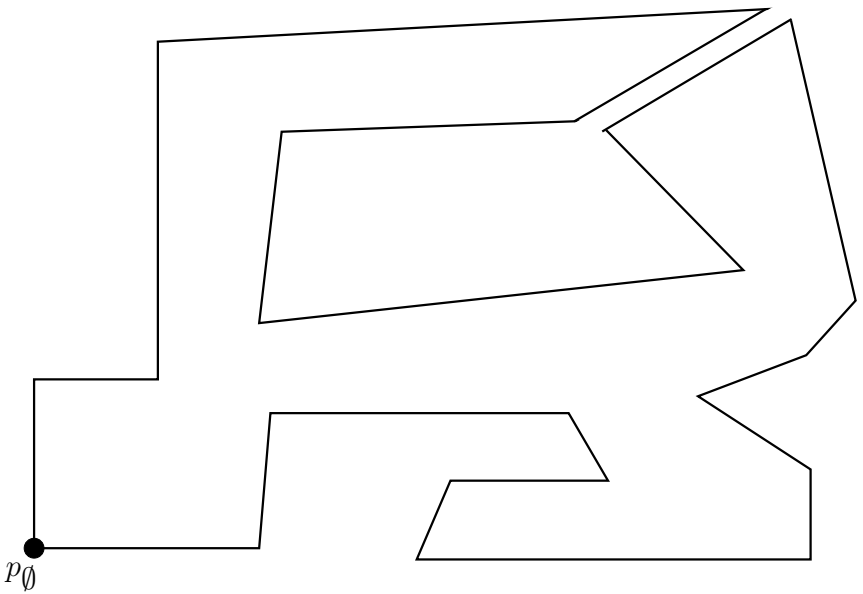


Parity-Based Vantage Point Selection Scheme

typical case: enumerate vertices of parent cell, choose odd vertex on g_ξ

Centralized Incremental Partition Algorithm

What happens with holes?



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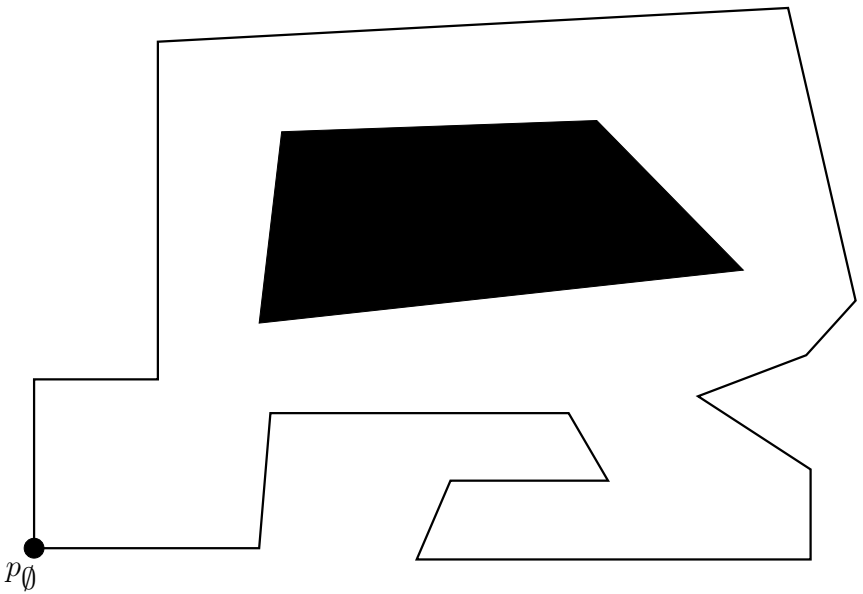

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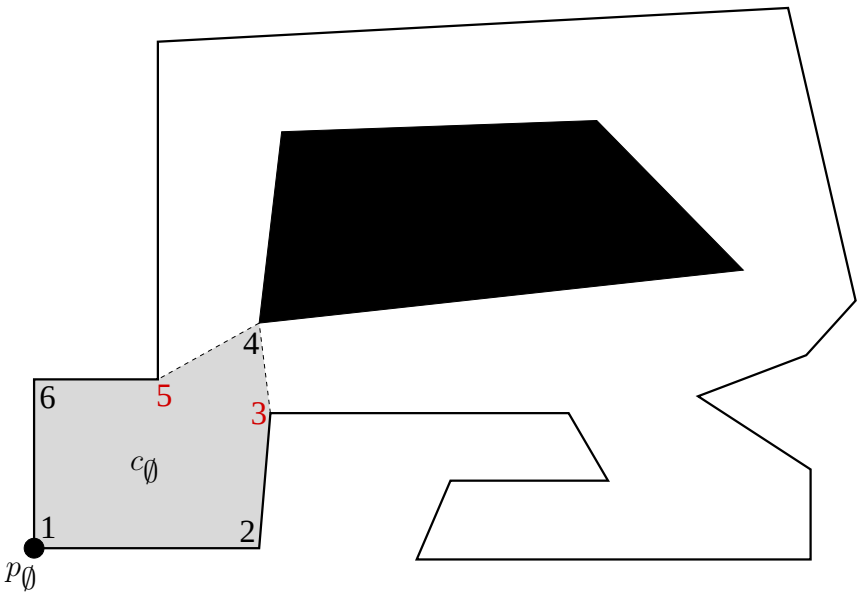

Partition Tree

$p_\emptyset, c_\emptyset$

Centralized Incremental Partition Algorithm

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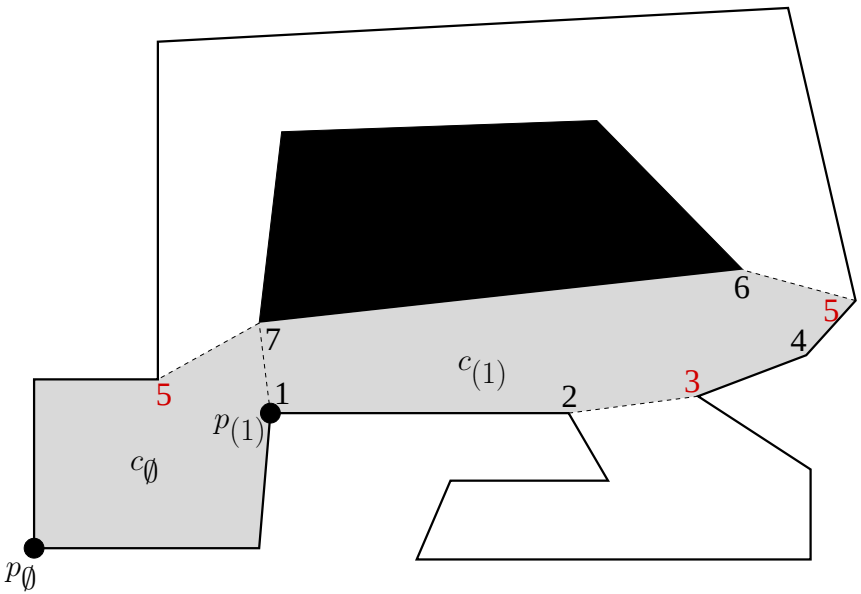

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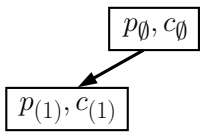

Centralized Incremental Partition Algorithm

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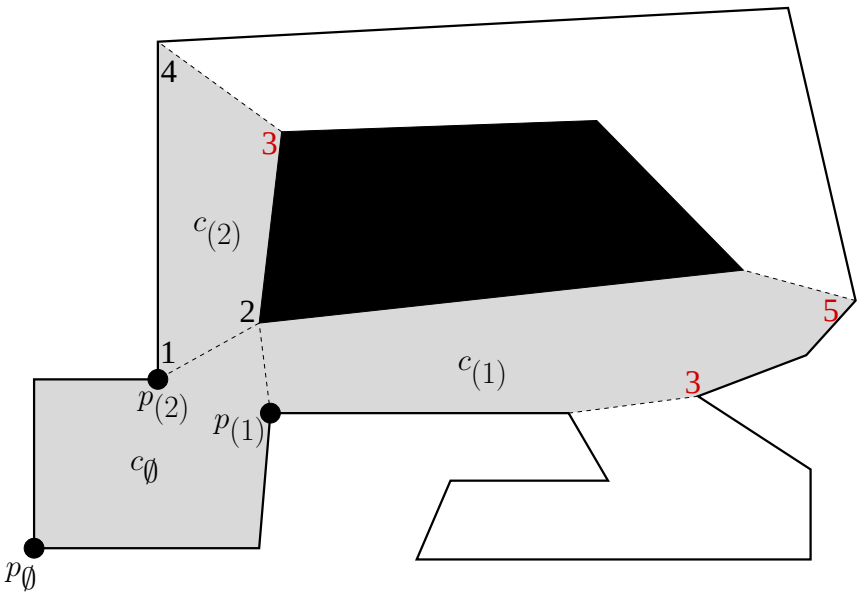

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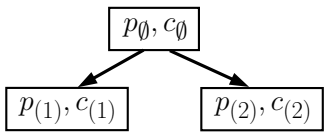

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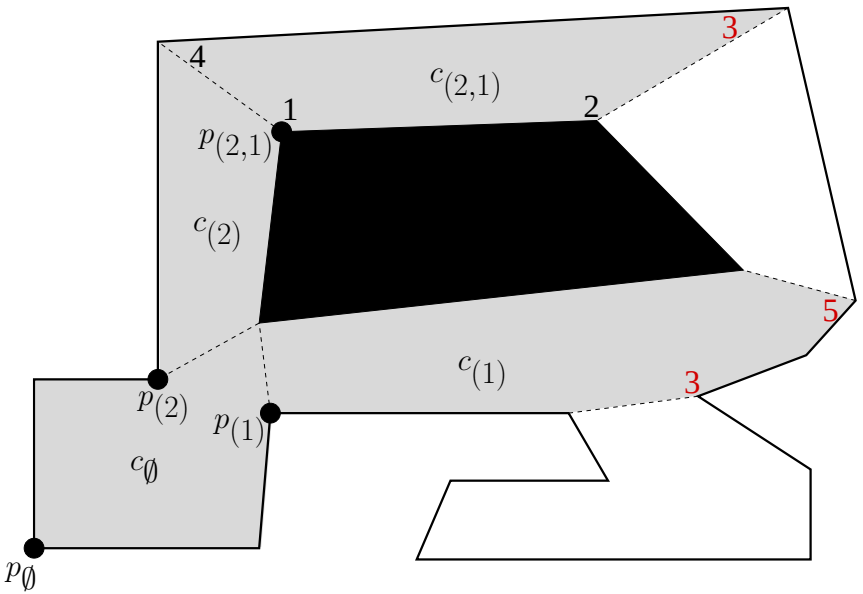

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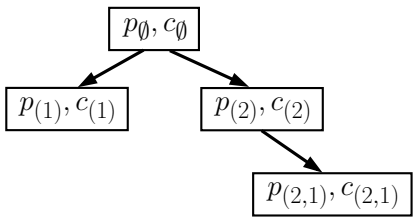

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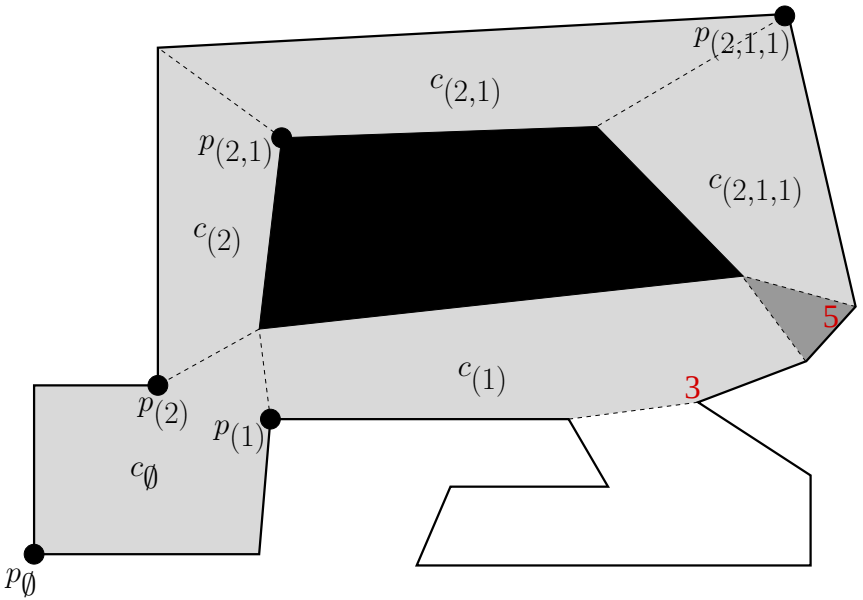

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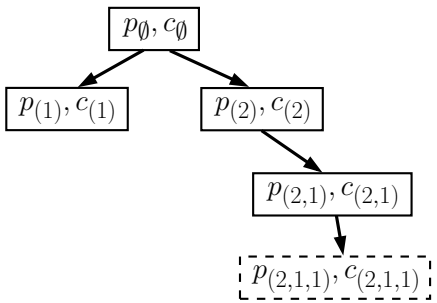

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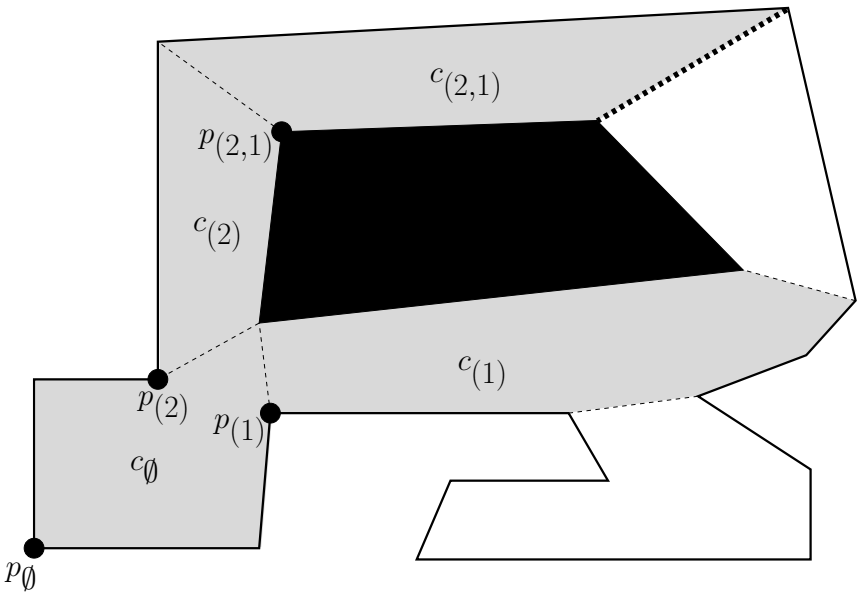

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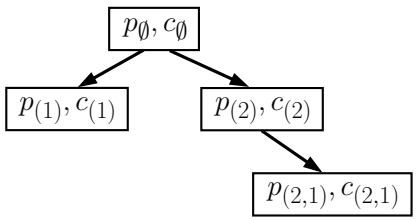

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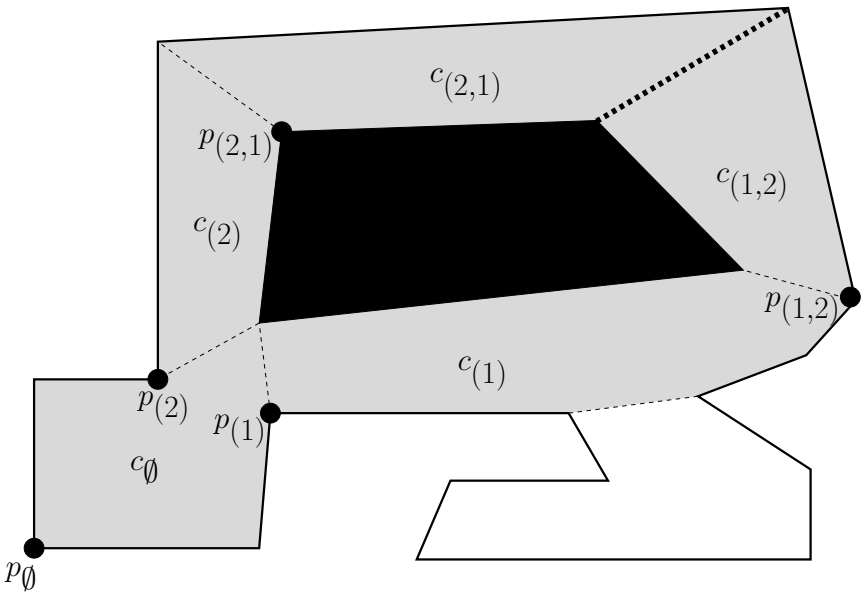

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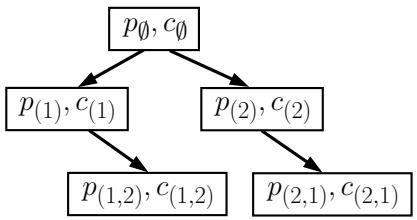

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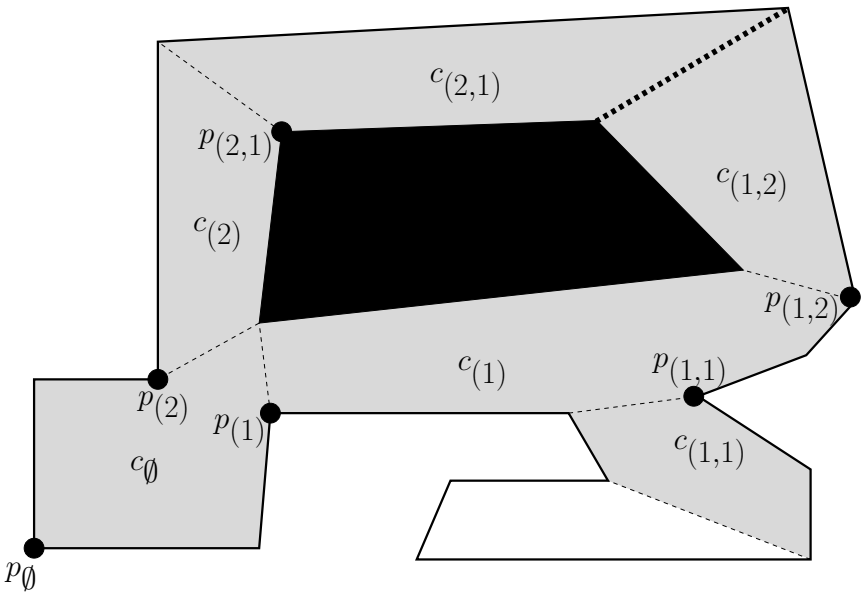

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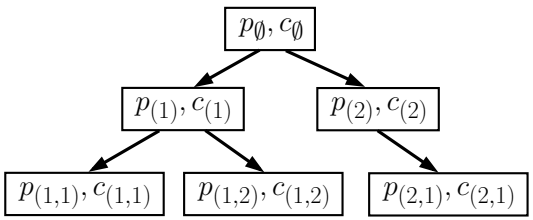

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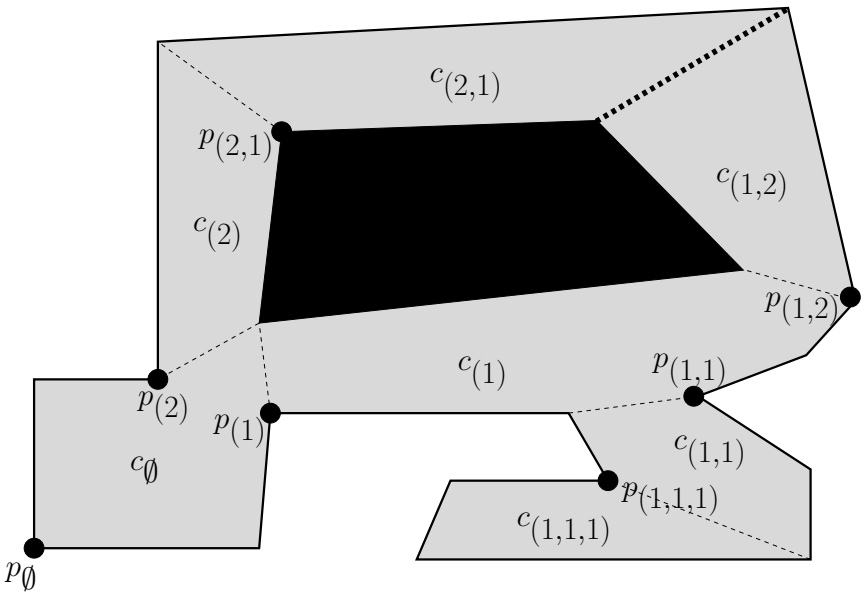

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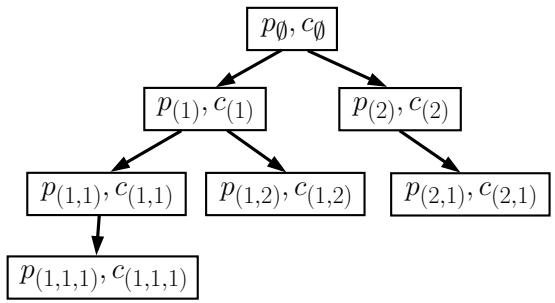

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if(c_ξ is in **branch conflict**)

discard vertex (p_ξ, c_ξ) and mark phantom wall at g_ξ;

else

insert vertex (p_ξ, c_ξ) into partition tree;

Convergence Theorem

The incremental partition algorithm

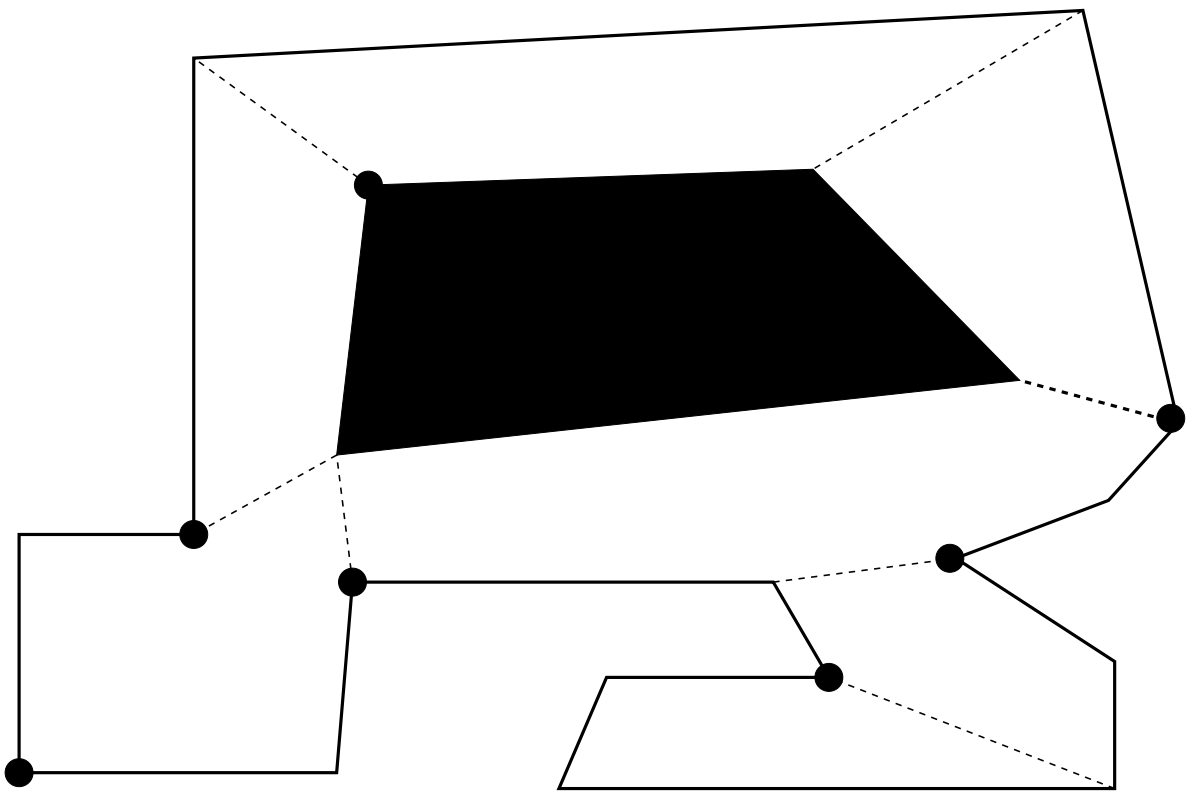
1. converges to full coverage in finite time,
2. the vantage points' visibility graph is connected, and
3. number of vantage points $N \leq \left\lfloor \frac{n+2h-1}{2} \right\rfloor$

Proof:

There are $n + 2h - 2$ triangles in any triangulation.

Centralized Incremental Partition Algorithm

Properties



Centralized Incremental Partition Algorithm

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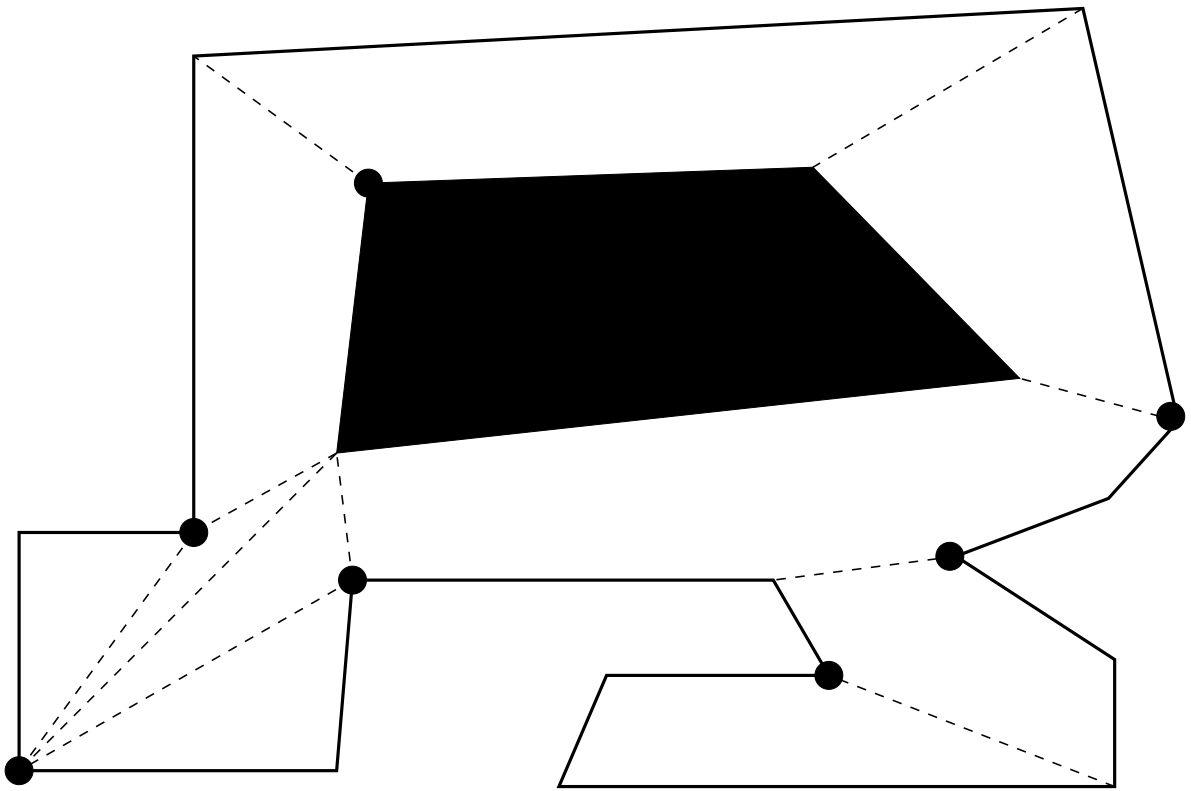

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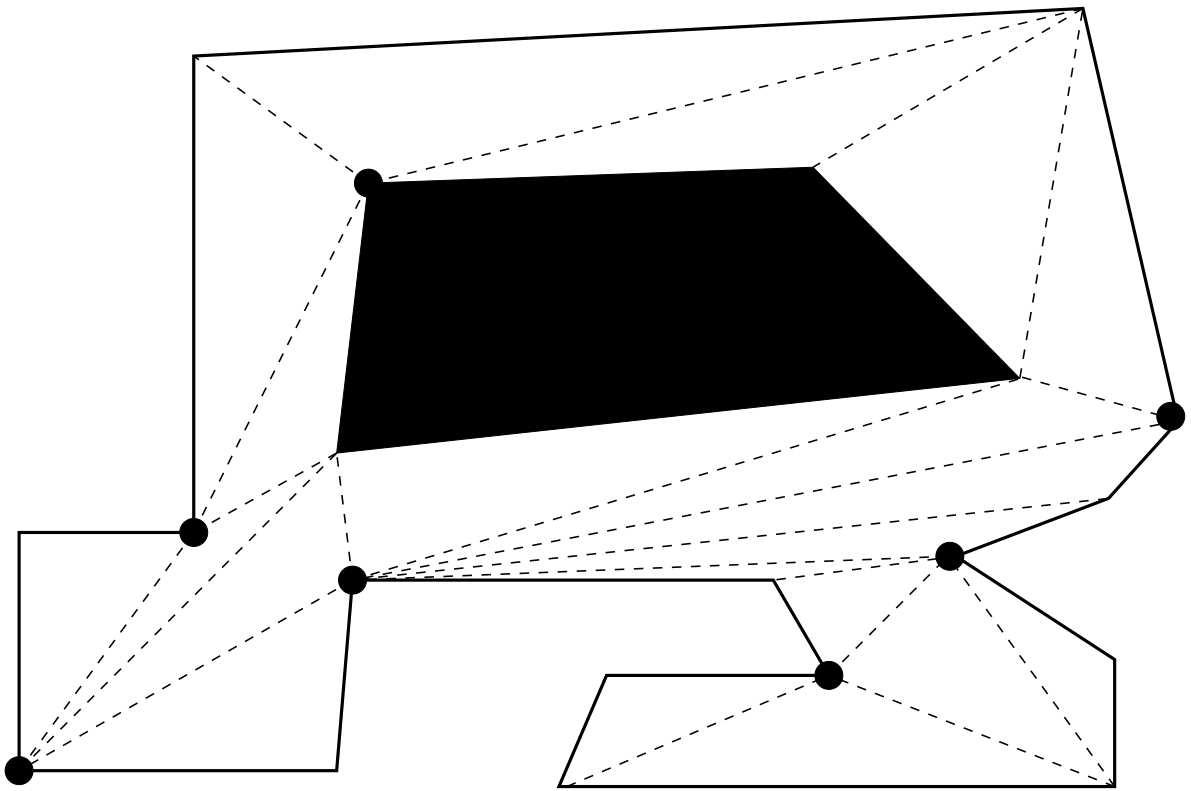

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Parity-based vantage point selection scheme $\Rightarrow$
Can assign 1 triangle to root vantage point,
2 triangles to other vantage points
$\Rightarrow 1 + 2(N - 1) \leq n + 2h - 2$

Depth-First Search

if(not all children visited)
    move to next child;
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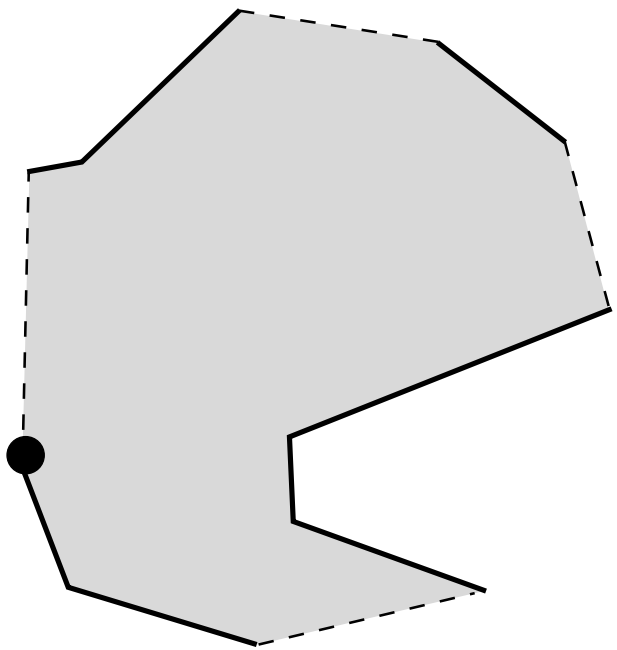

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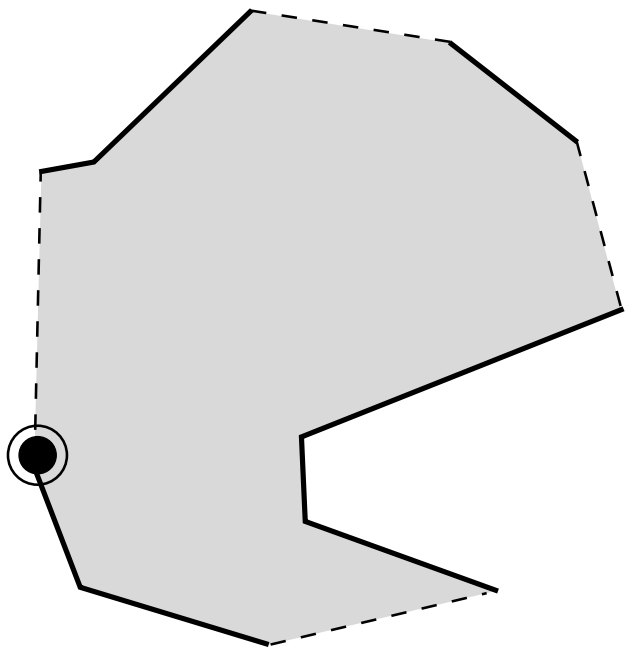

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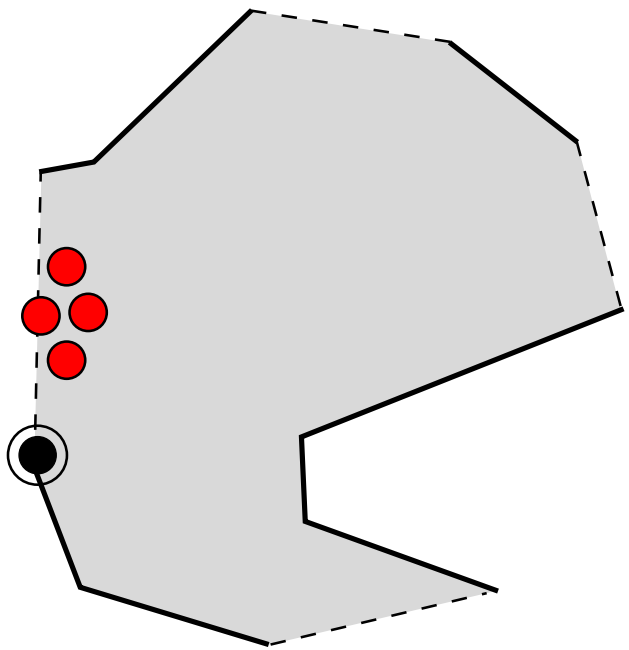

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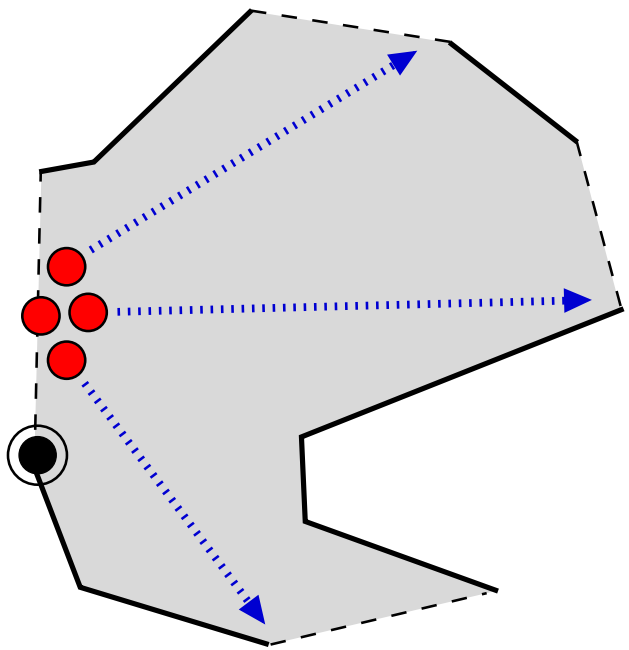

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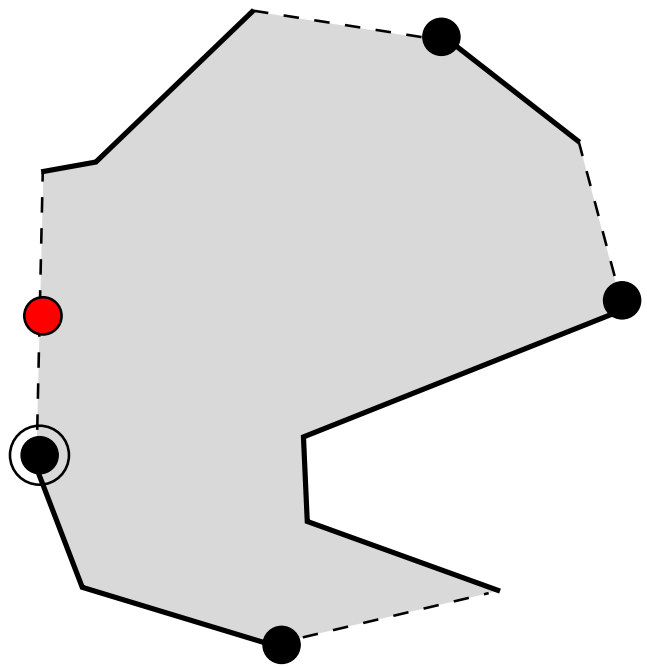

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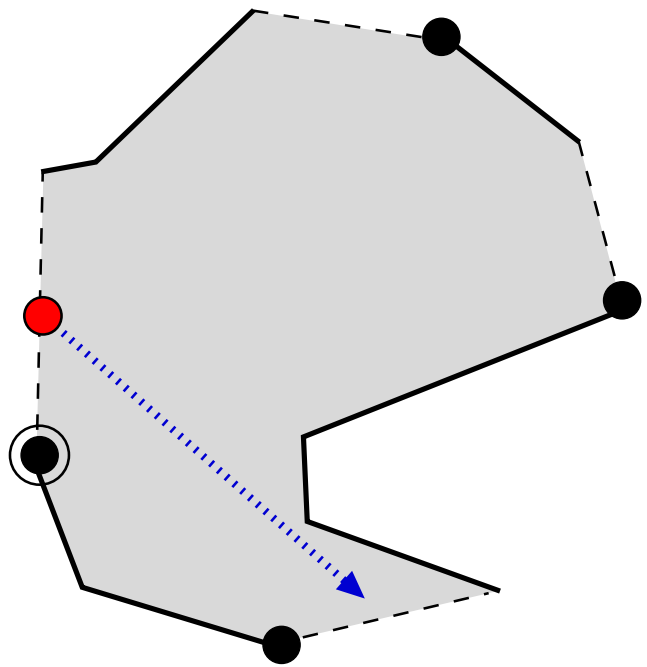

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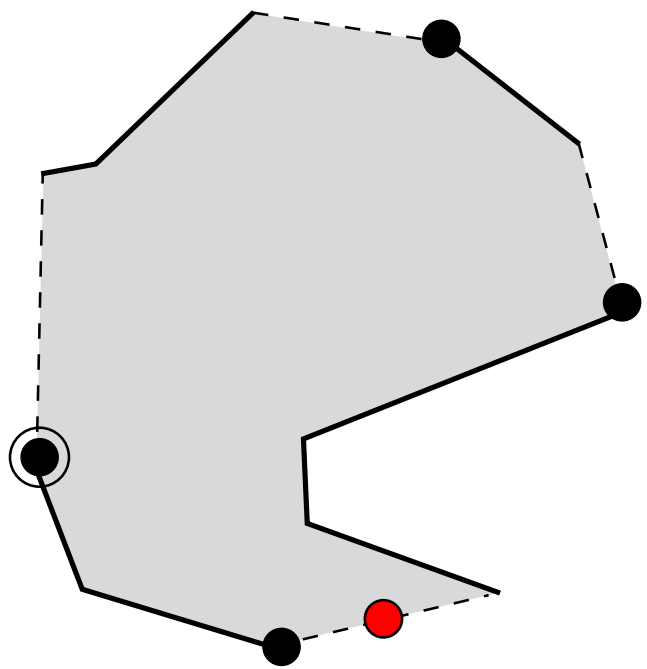

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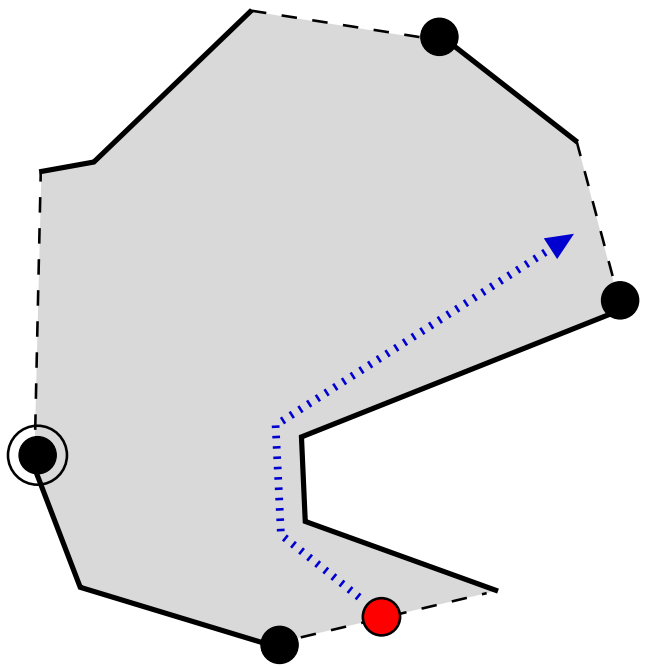

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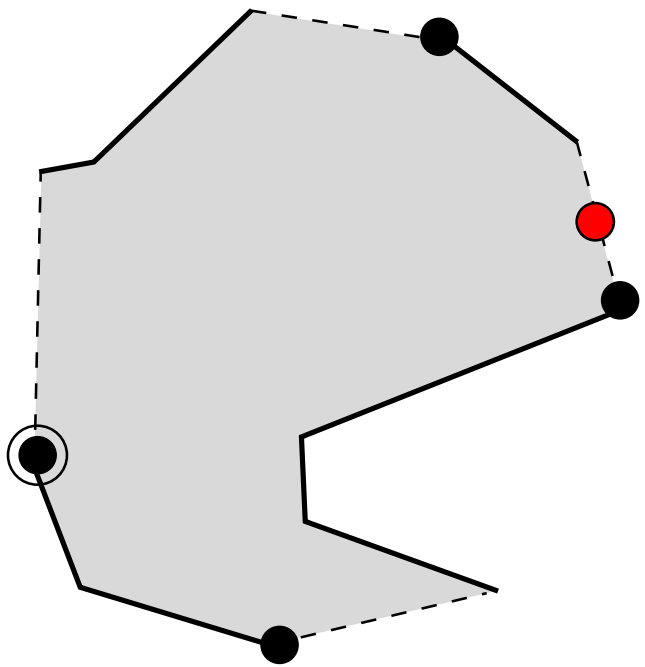

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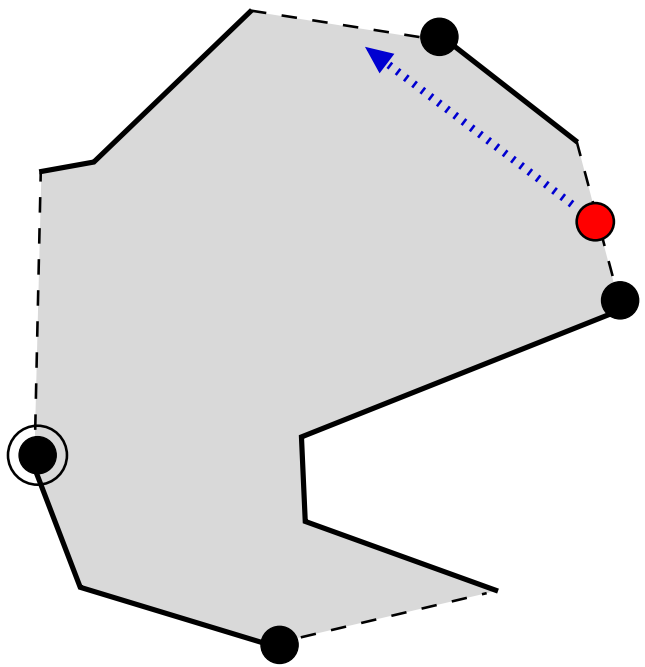

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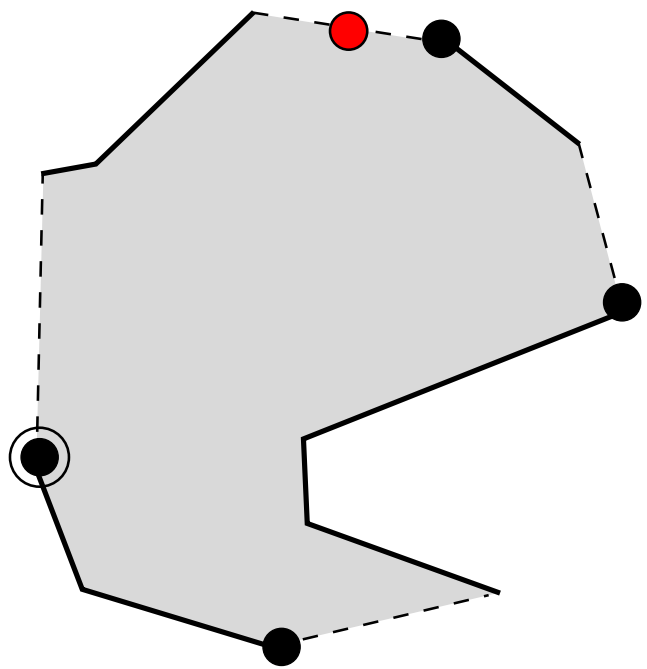

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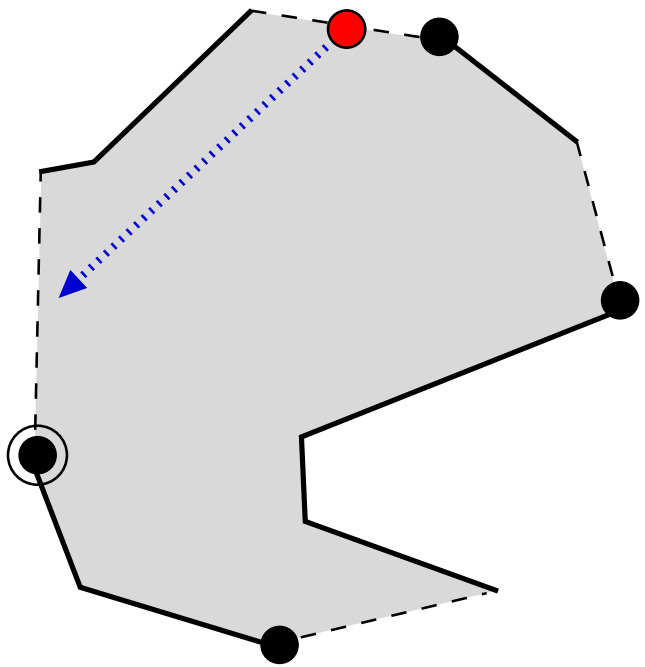

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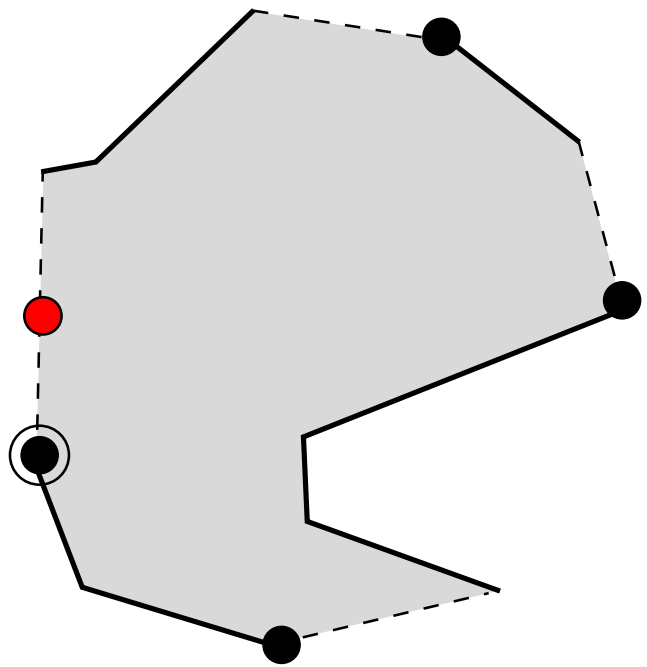

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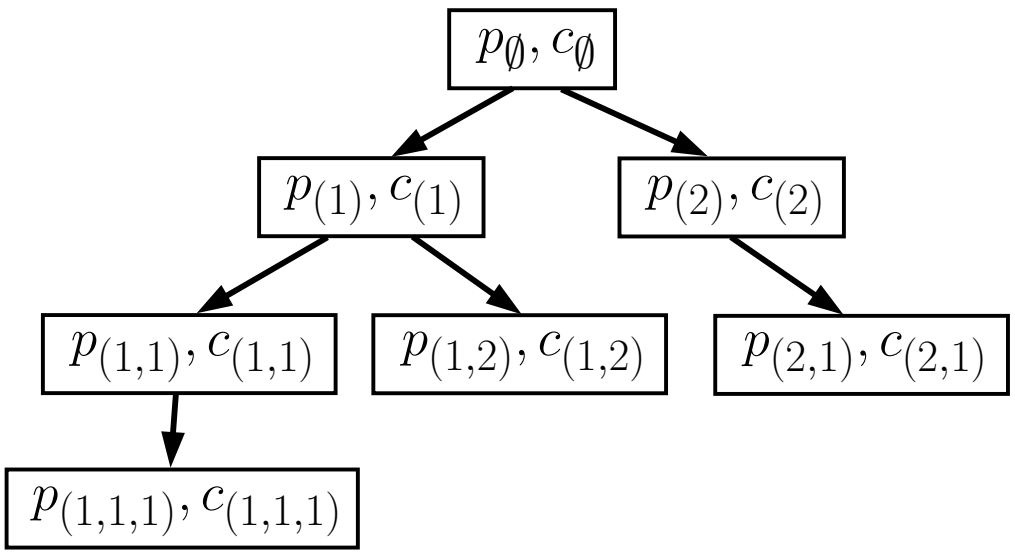

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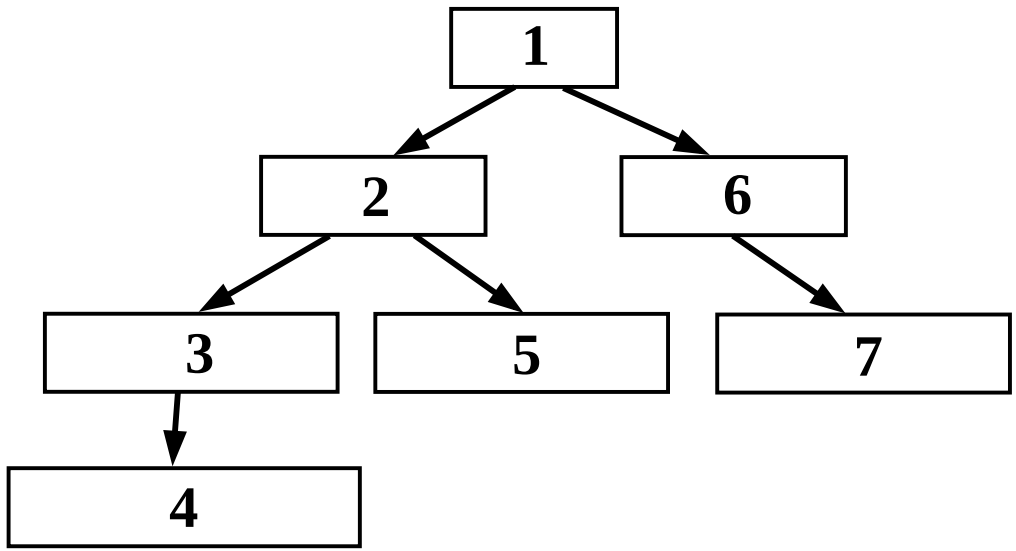

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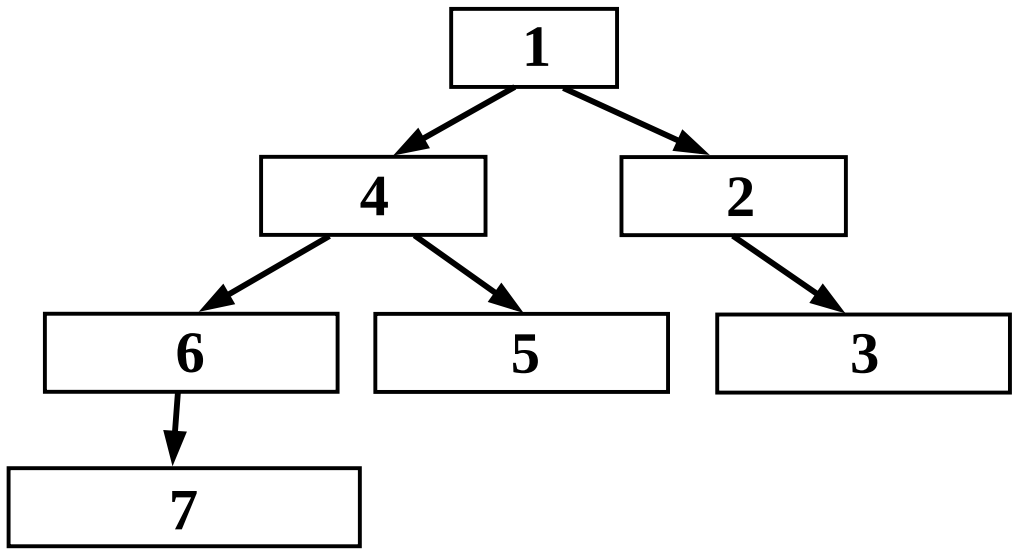

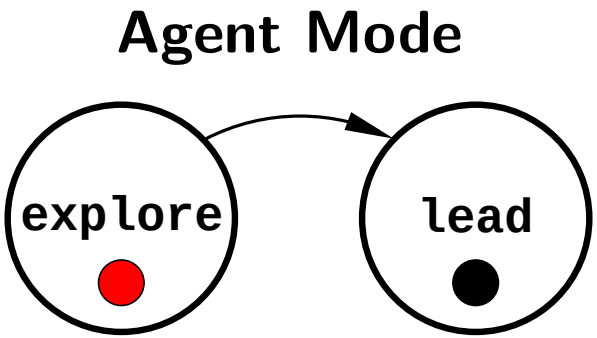
Agent Mode
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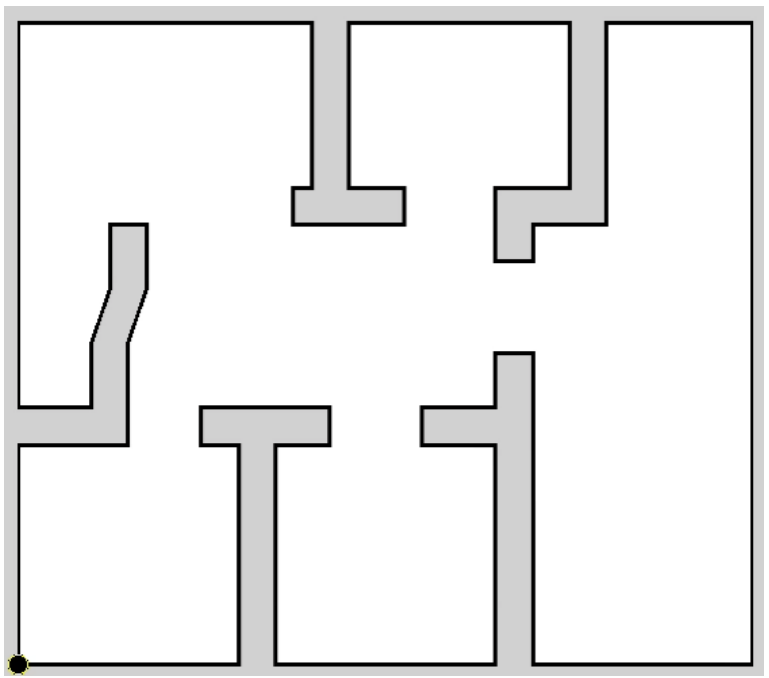

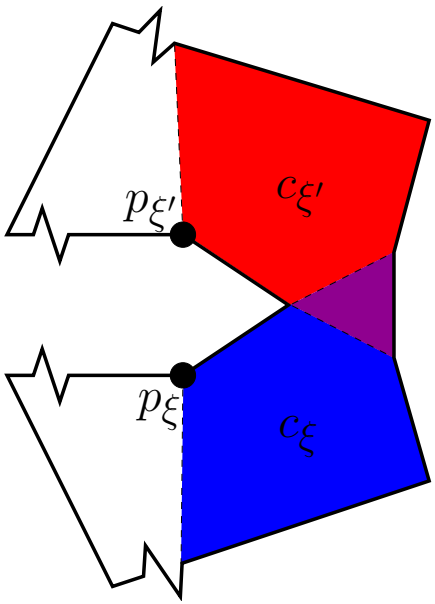
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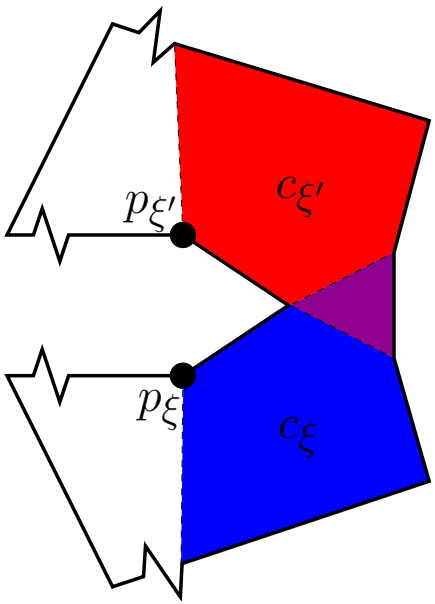
$c_{\xi'}$
p_{ξ}
c_{ξ}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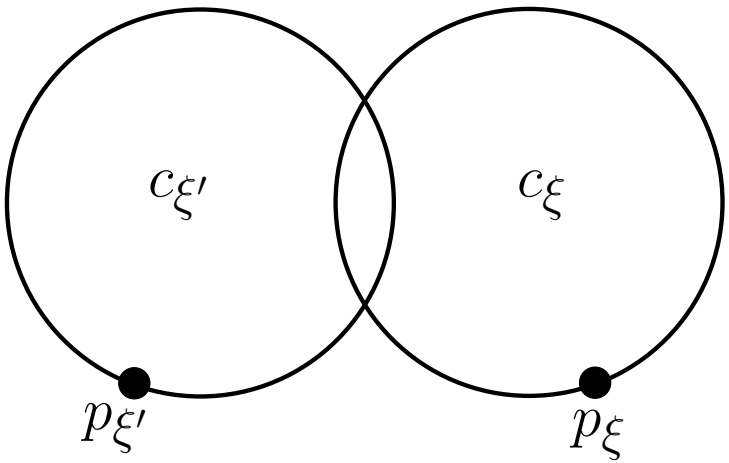
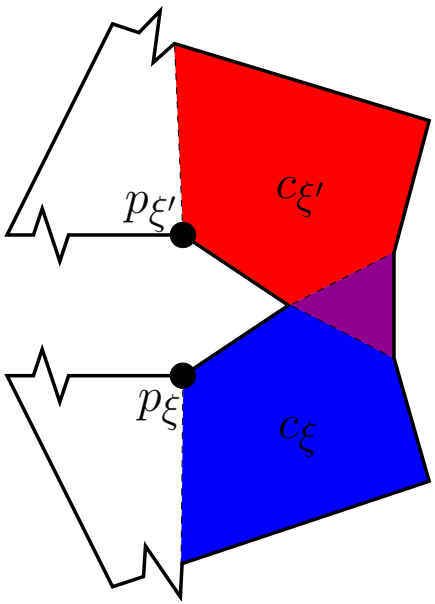
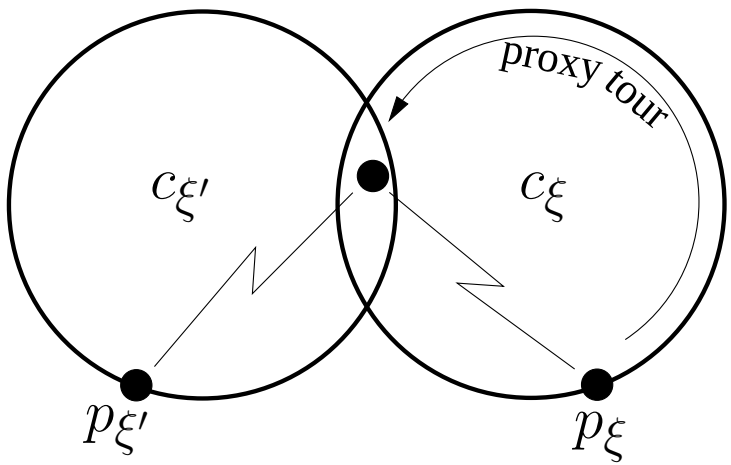
p_{ξ}
c_{ξ}


$c_{\xi'}$
c_{ξ}
$p_{\xi'}$
p_{ξ}

Distributed Deployment Algorithm

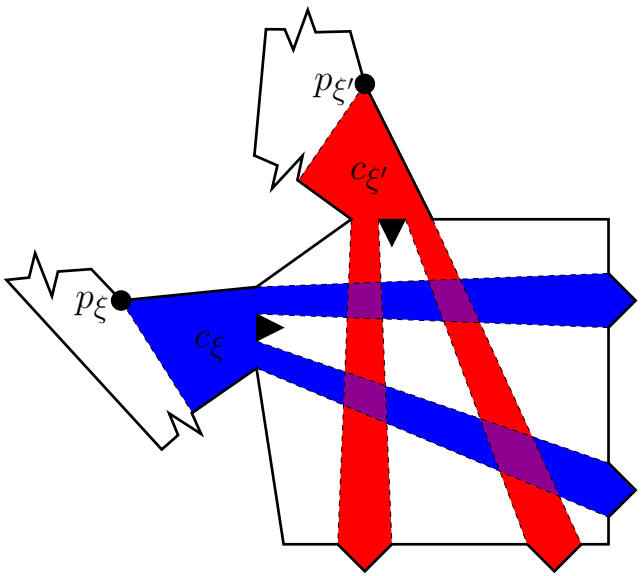
Branch Deconfliction via Proxy Agents

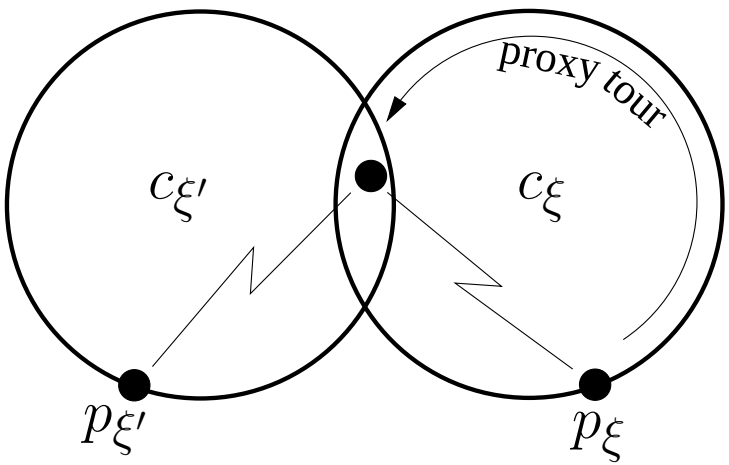


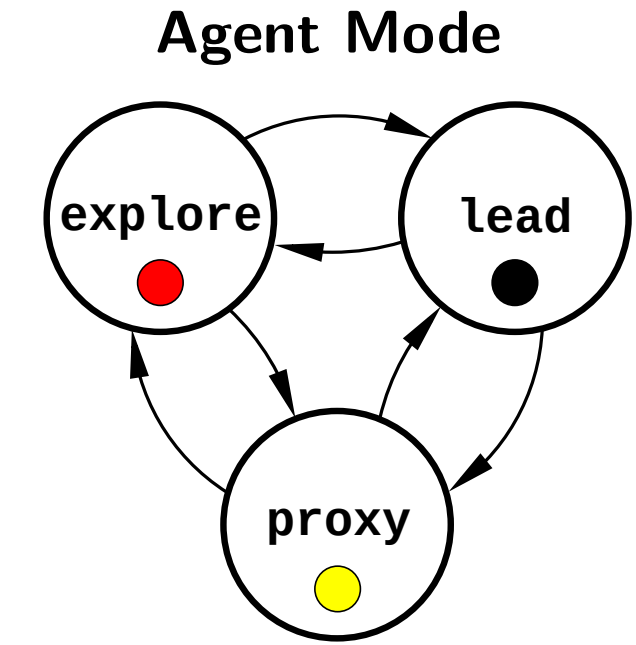


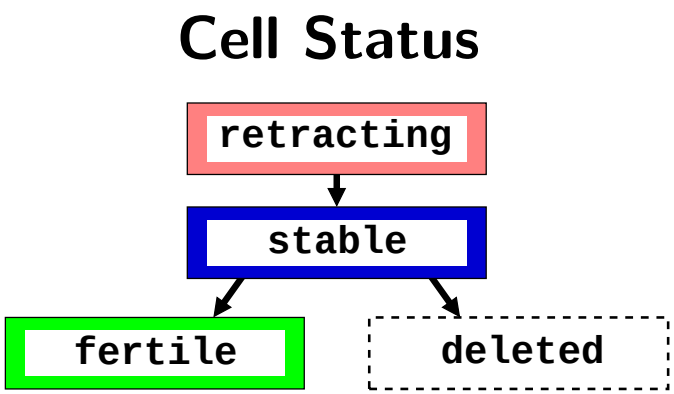
Distributed Deployment Algorithm

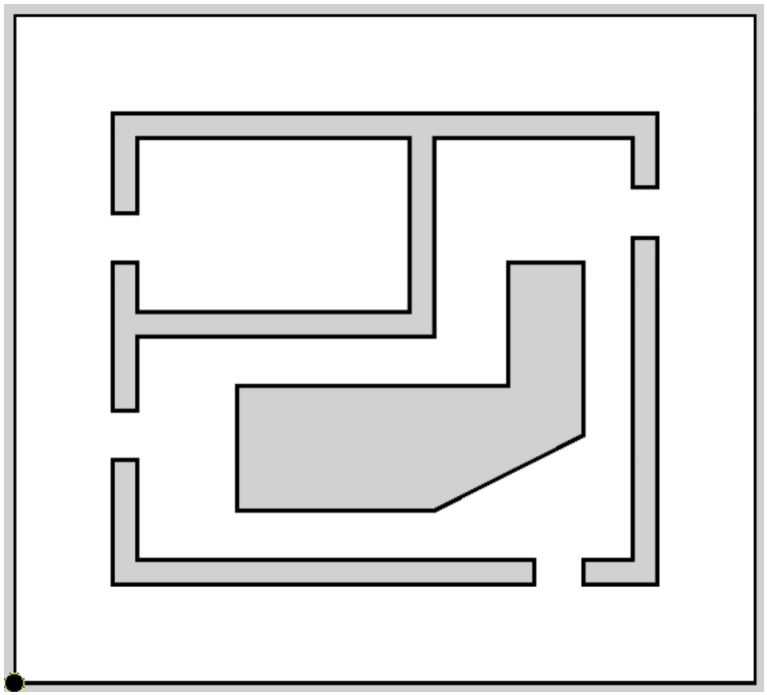
Branch Deconfliction via Proxy Agents






Agent Mode
explore
lead
proxy


Cell Status
retracting
stable
fertile
deleted



Convergence Theorem

If N agents operate according to the Distributed Deployment Algorithm, then

1. the agents' visibility graph remains connected,
2. there exists a finite time t^*, such that the partition tree remains unchanged for all $t > t^*$, and
3. if $N \geq \lfloor \frac{n+2h-1}{2} \rfloor$, then for all $t > t^*$ there is full coverage

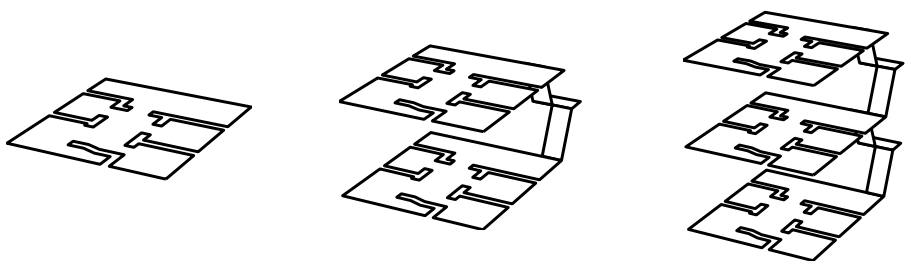
Theorem(Time to Converge)
Assuming uniformly bounded environment diameter, time until full coverage is $\mathcal{O}(n^2 + nh)$. If max cell perimeter uniformly bounded, then $\mathcal{O}(n + h)$

Lemma(Memory and Message Size)
Let k:= max number of vertices of any vertex-limited visibility polygon. Then an agent

1. requires $\mathcal{O}(Nk)$ bits of memory, and
2. sends messages of size $\mathcal{O}(k)$ bits

Extensions

- Robustness to agent failure
- Robustness to opening doors
- Multiple roots
- Combining with distributed assignment
- 2.5D environments



Part III
UAV Path Planning

Visual Reconnaissance Problem

Goal

Fly a camera-equipped UAV over a terrain such that a set of ground targets can be photographed in minimum time

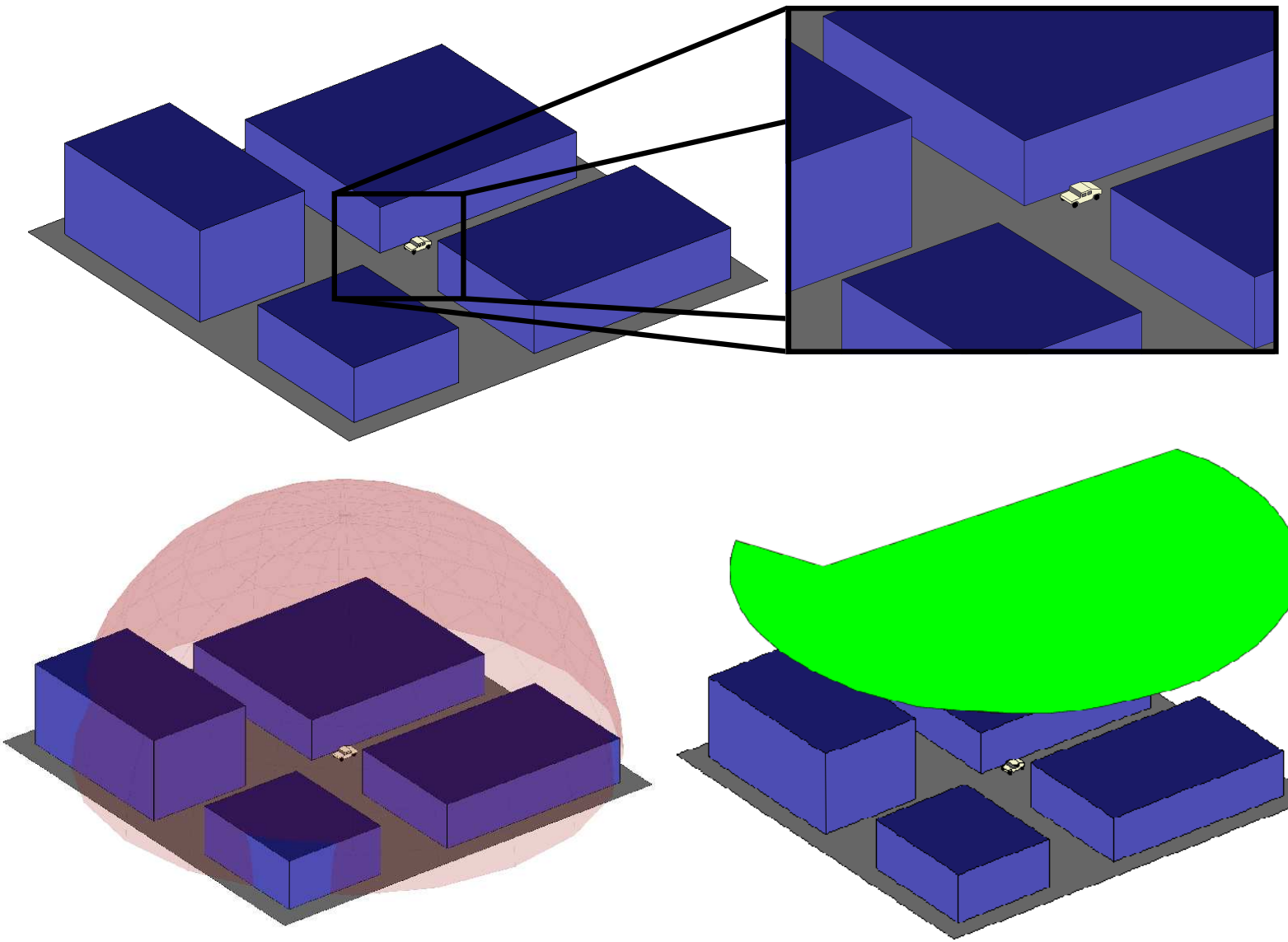
Assumptions

- fixed-wing UAV
- constant altitude, constant airspeed
- static terrain, static targets
- fast* gimbaled camera

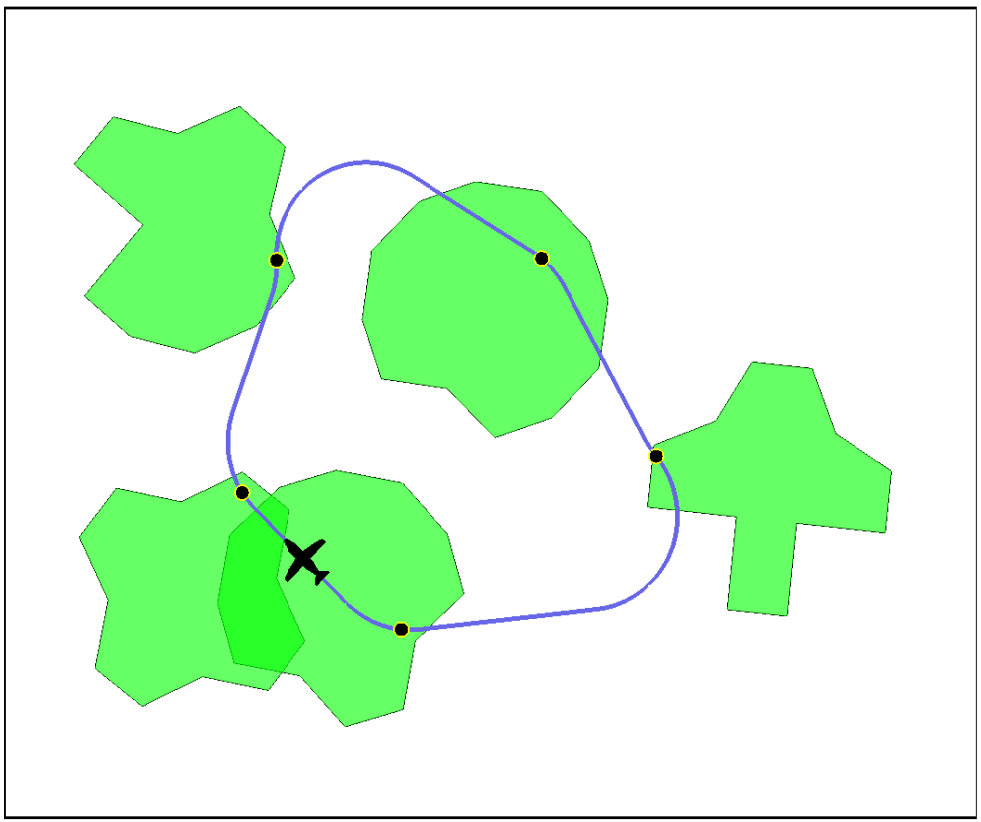
*Gimbal dynamics much faster than the UAV dynamics

Visibility of a Target in Terrain

Intersecting **Range**, **Altitude**, and **Occlusion** Constraints

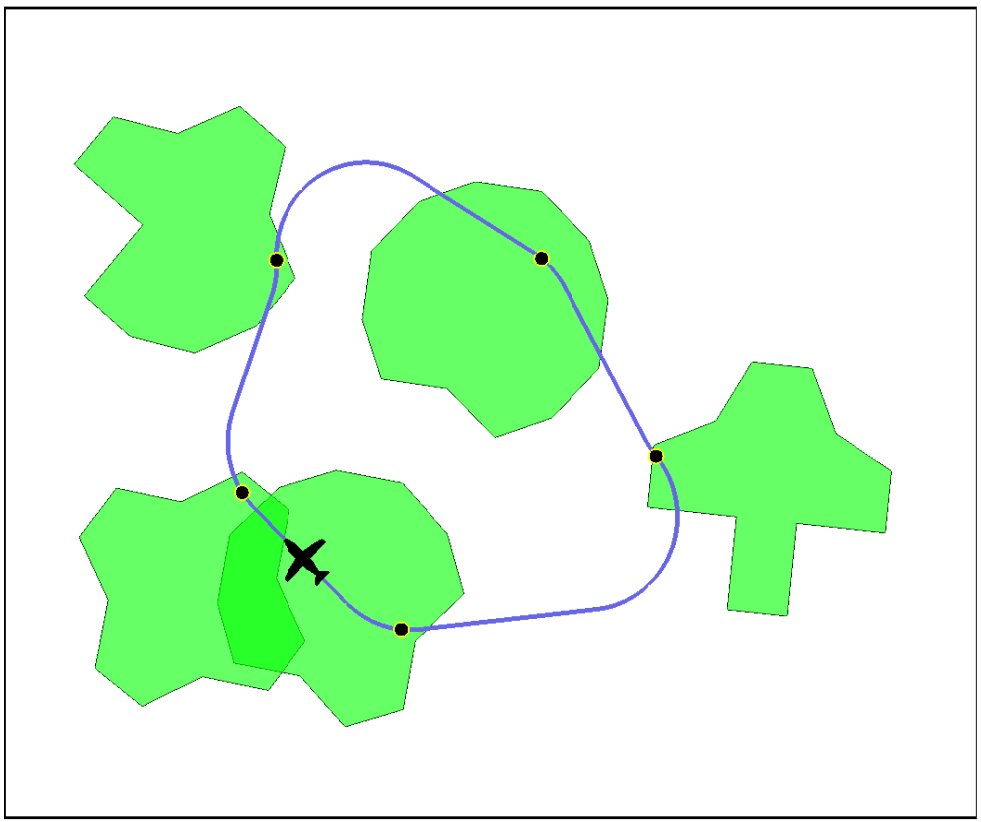


Polygon-Visiting Dubins TSP (PVDTSP)



Polygon-Visiting Dubins TSP (PVDTSP)

$$\left(\begin{bmatrix} x_1 \\ y_1 \\ \psi_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \\ \psi_2 \end{bmatrix}, \begin{bmatrix} x_3 \\ y_3 \\ \psi_3 \end{bmatrix}, \begin{bmatrix} x_4 \\ y_4 \\ \psi_4 \end{bmatrix}, \begin{bmatrix} x_5 \\ y_5 \\ \psi_5 \end{bmatrix} \right)$$



Vehicle Dynamics

$$\mathbf{x} = (x, y, \psi) \in X = \mathbb{R}^2 \times \mathbb{S} = \mathrm{SE}(2)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} V_{\mathrm{a}} \sin(\psi) \\ V_{\mathrm{a}} \cos(\psi) \\ u \end{bmatrix}, \quad u \leq u_{\max} \quad (\text{max turn rate})$$

Optimization Problem

$$\begin{aligned} \text{Minimize :} \quad & C(\mathbf{x}_1, \ldots, \mathbf{x}_n) = \textstyle\sum_{i=1}^{n-1} d(\mathbf{x}_i, \mathbf{x}_{i+1}) + d(\mathbf{x}_n, \mathbf{x}_1) \\ \text{Subject To :} \quad & \text{for each } i \text{ there exists } j \text{ s.t. } \mathbf{x}_j \in \mathcal{V}(\mathcal{T}_i) \end{aligned}$$

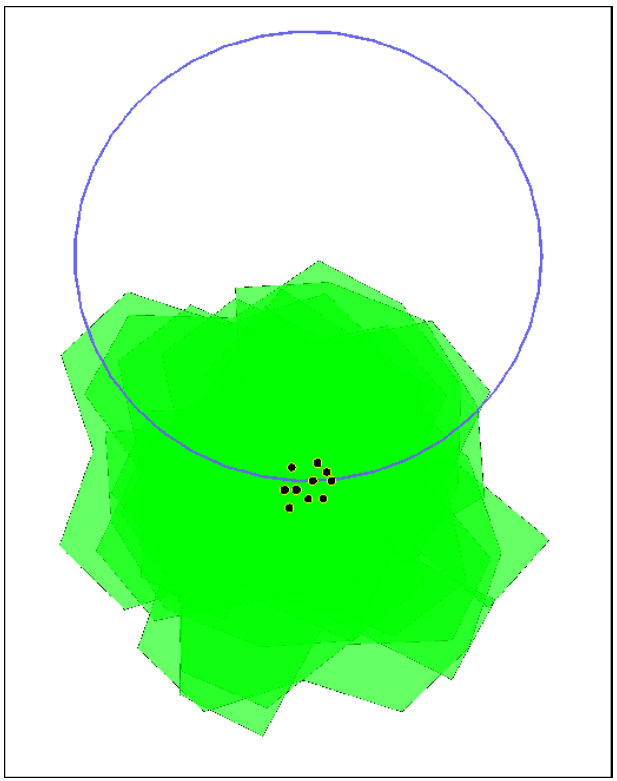
target visibility set $\mathcal{V}(\mathcal{T}_i) \subset X$

state-to-state cost from BVP solver

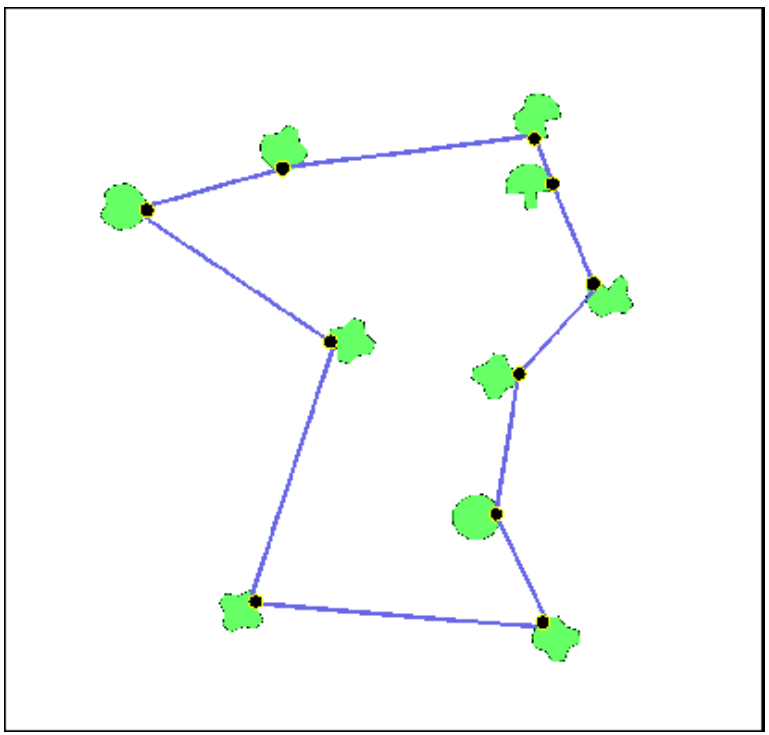
Theorem

Solving DTSP instead of PVDTSP $\Rightarrow$ $\Omega(n)$ penalty in worst case

Dense Limit



Sparse Limit

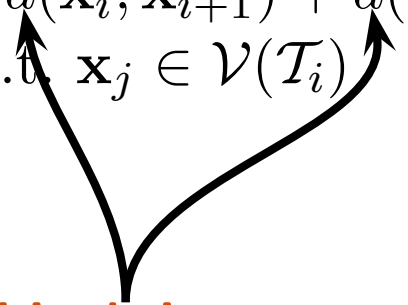


Algorithm Design Requirements

- **Provable convergence**
- **Speed** suitable for online purposes
- **Ability to trade off computation time** for solution quality
- **Extensibility** for
  - Wind
  - Airspace constraints
  - Any vehicle dynamics

Optimization Problem

$$\begin{aligned} \text{Minimize}: \quad & C(\mathbf{x}_1, \ldots, \mathbf{x}_n) = \sum_{i=1}^{n-1} d(\mathbf{x}_i, \mathbf{x}_{i+1}) + d(\mathbf{x}_n, \mathbf{x}_1) \\ \text{Subject To}: \quad & \text{for each } i \text{ there exists } j \text{ s.t. } \mathbf{x}_j \in \mathcal{V}(\mathcal{T}_i) \end{aligned}$$



treating state-to-state distance function as **black box**

$\Rightarrow$ can use any BVP solver which handles

- Wind
- Airspace constraints
- Any vehicle dynamics

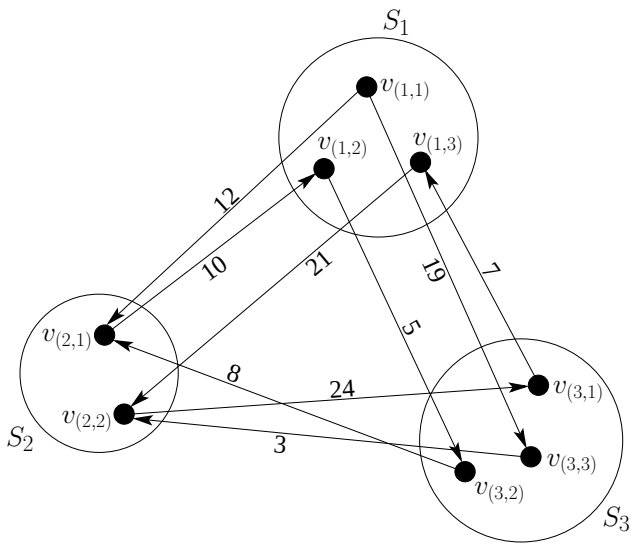
ATSP (Asymmetric TSP)

In a weighted directed graph, find a minimum weight closed tour which visits each vertex exactly once

- NP-Hard, but state-of-the-art heuristics are effectively exact and have $\mathcal{O}(n^{2.2})$ average case runtime [Helsgaun, 2000]

FOTSP (Finite One-in-a-set TSP)

In a weighted directed graph with vertices partitioned into clusters, find a minimum weight closed tour which visits at least one vertex in each cluster



Relevant Variations of the TSP

ATSP (Asymmetric TSP)

In a weighted directed graph, find a minimum weight closed tour which visits each vertex exactly once

- NP-Hard, but state-of-the-art heuristics are effectively exact and have $\mathcal{O}(n^{2.2})$ average case runtime [Helsgaun, 2000]

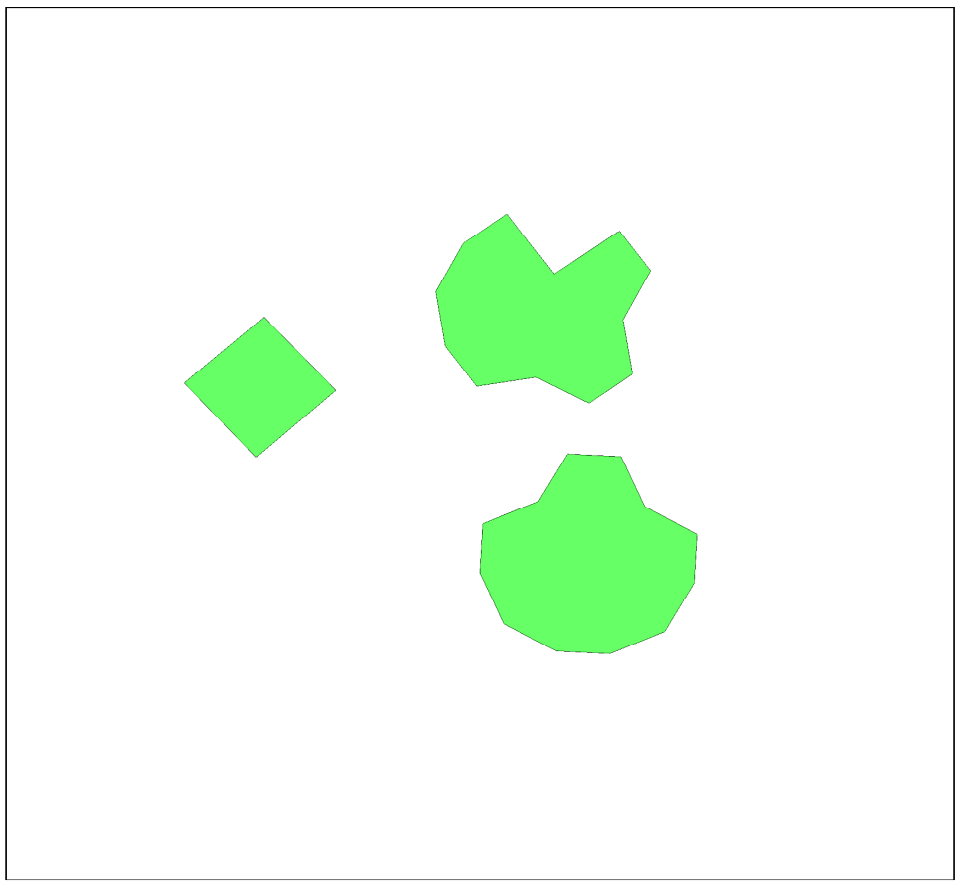
FOTSP (Finite One-in-a-set TSP)

In a weighted directed graph with vertices partitioned into clusters, find a minimum weight closed tour which visits at least one vertex in each cluster

- Can be solved exactly via Noon-Bean transformation [Noon and Bean, 1991]

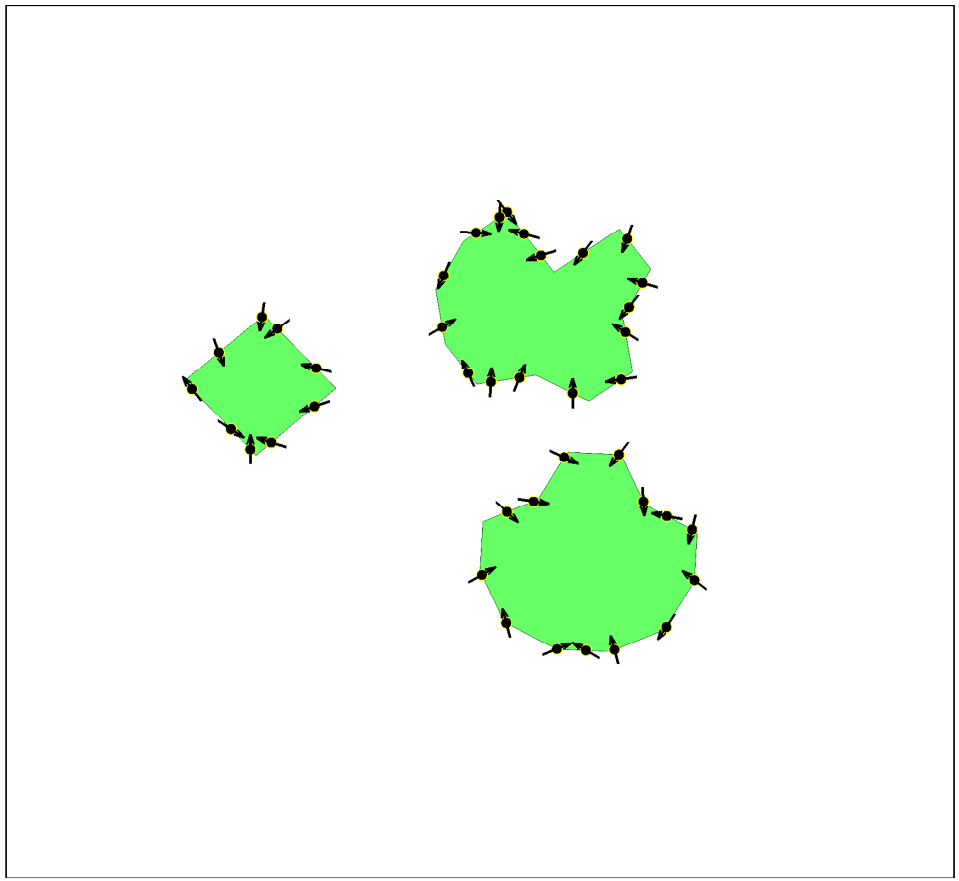
PVDTSP Roadmap Construction

(FOTSP instance)



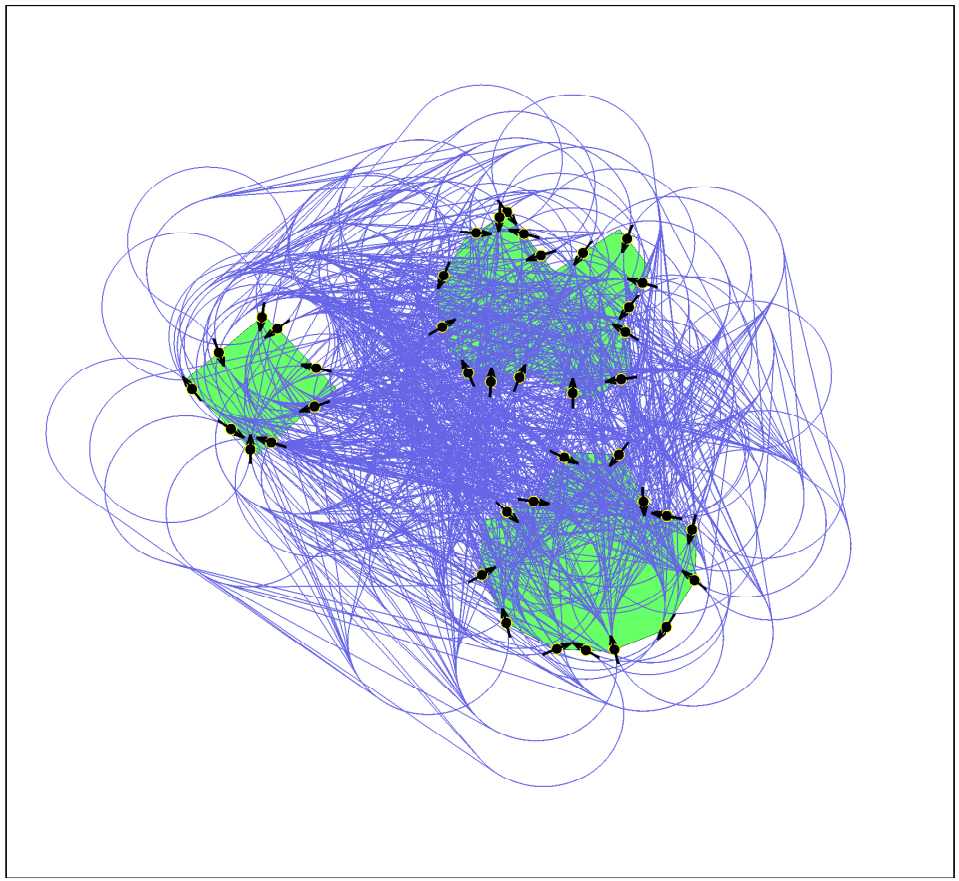
PVDTSP Roadmap Construction

(FOTSP instance)

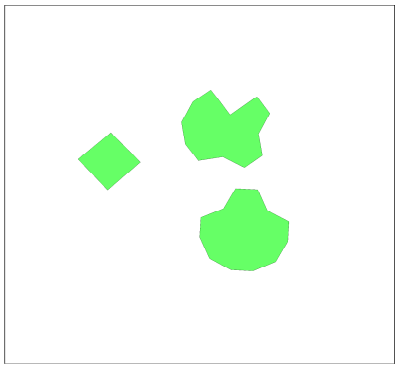


PVDTSP Roadmap Construction

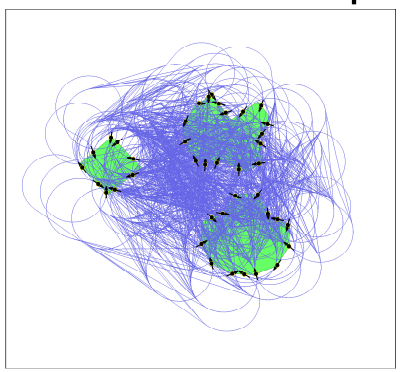
(FOTSP instance)



Input
PVDTSP Instance

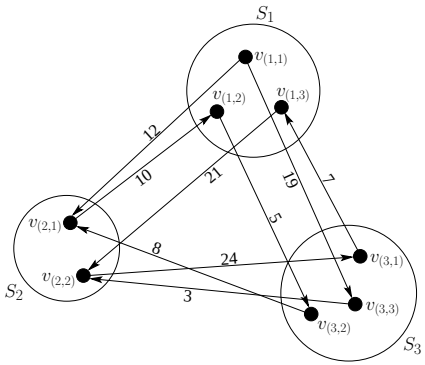


Build Roadmap

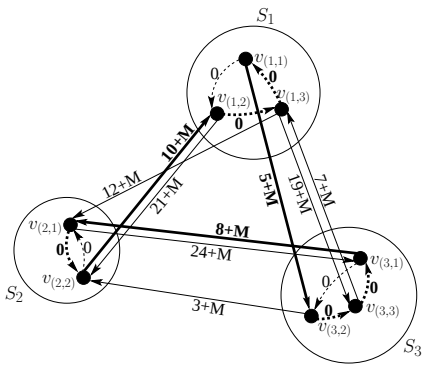


=

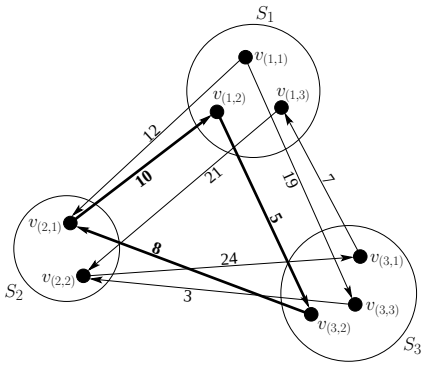
FOTSP Instance



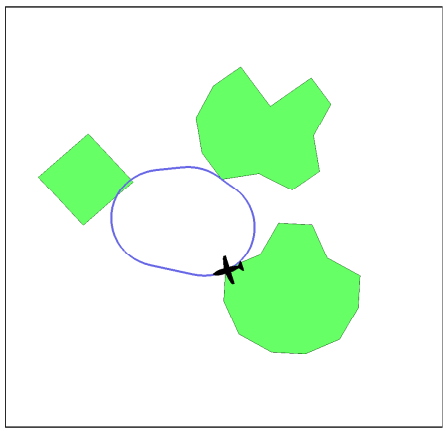
Build + Solve
ATSP Instance
(Noon-Bean Transf.)



Extract
FOTSP Solution



Return
Final Path



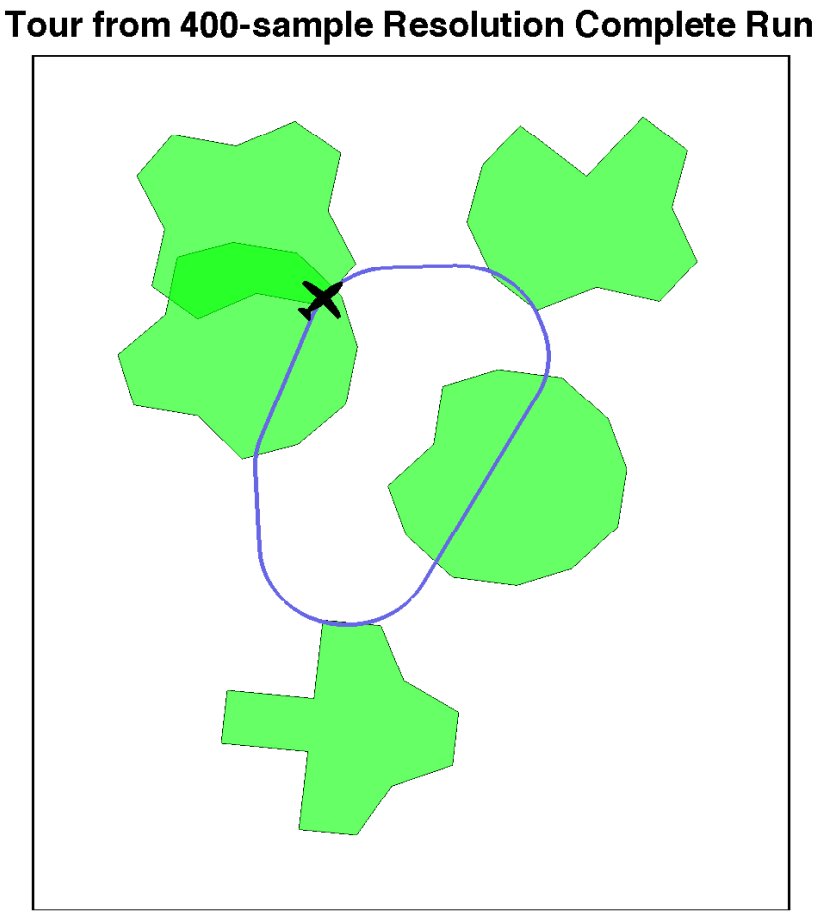
Theorem (Resolution Completeness)
Let $\{\tau_i\}_{i=1}^{\infty}$ be the sequence of tours computed by the sampling-based roadmap method when applied to the sequence of roadmaps $\{\mathcal{R}_i\}_{i=1}^{\infty}$, respectively. Then $\{C(\tau_i)\}_{i=1}^{\infty}$ is nonincreasing and

$$\lim_{i\to\infty} C(\tau_i) \leq \inf_{\tau\in\mathcal{D}^{\circ}_{\text{feas}}} C(\tau).$$

- empirically determined average case time complexity* $\mathcal{O}(n_{\text{samples}}^{2.2})$

Numerical Study

5 Targets, ~ 10 sec. in C++ on 2.33 GHz i686



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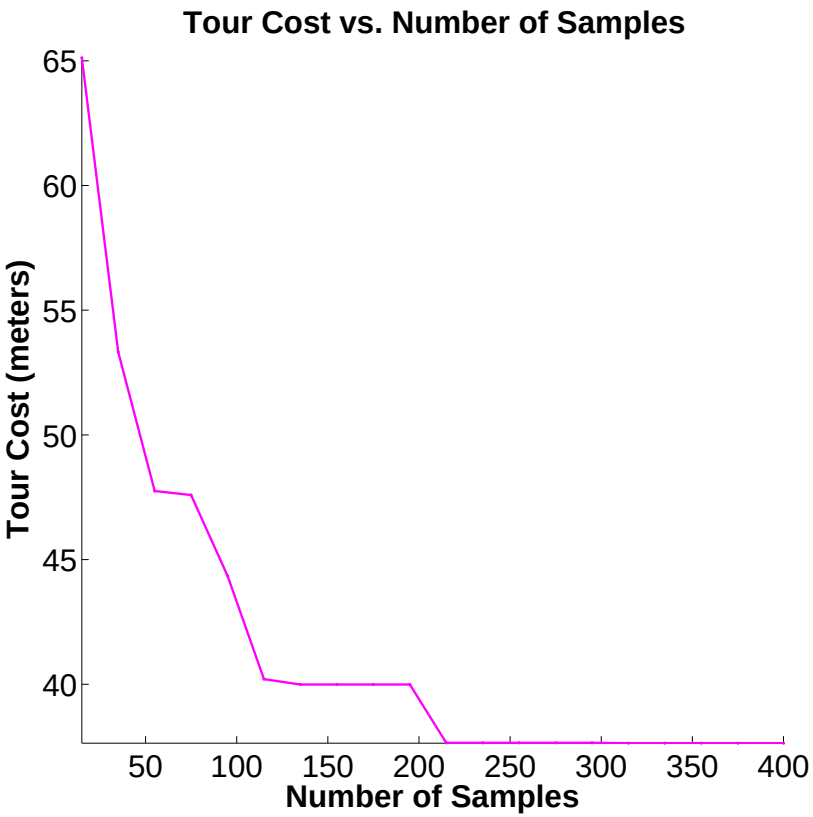
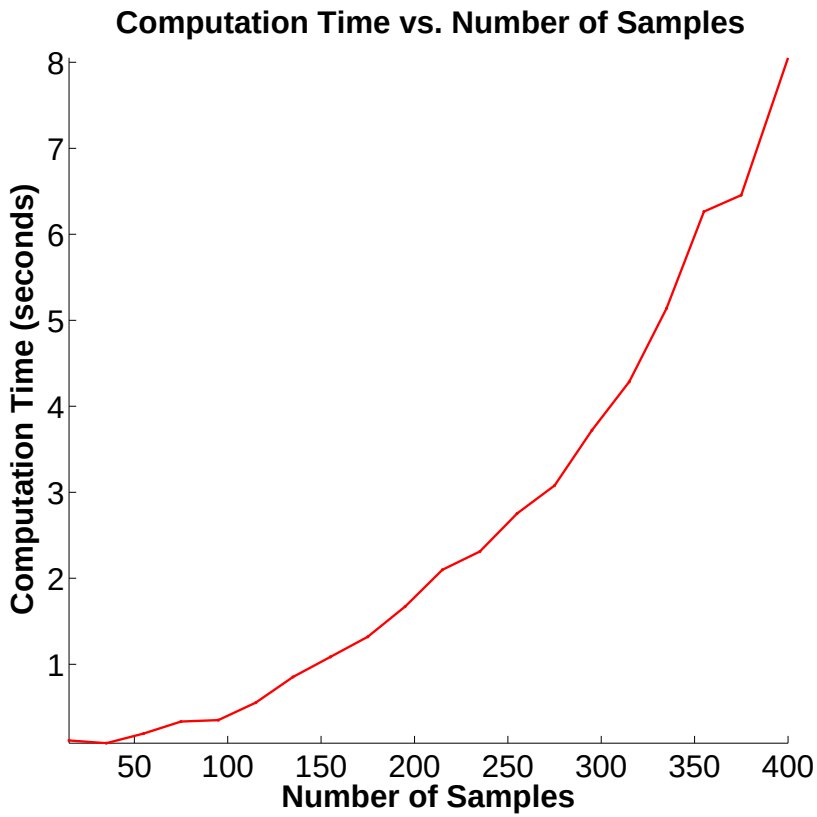
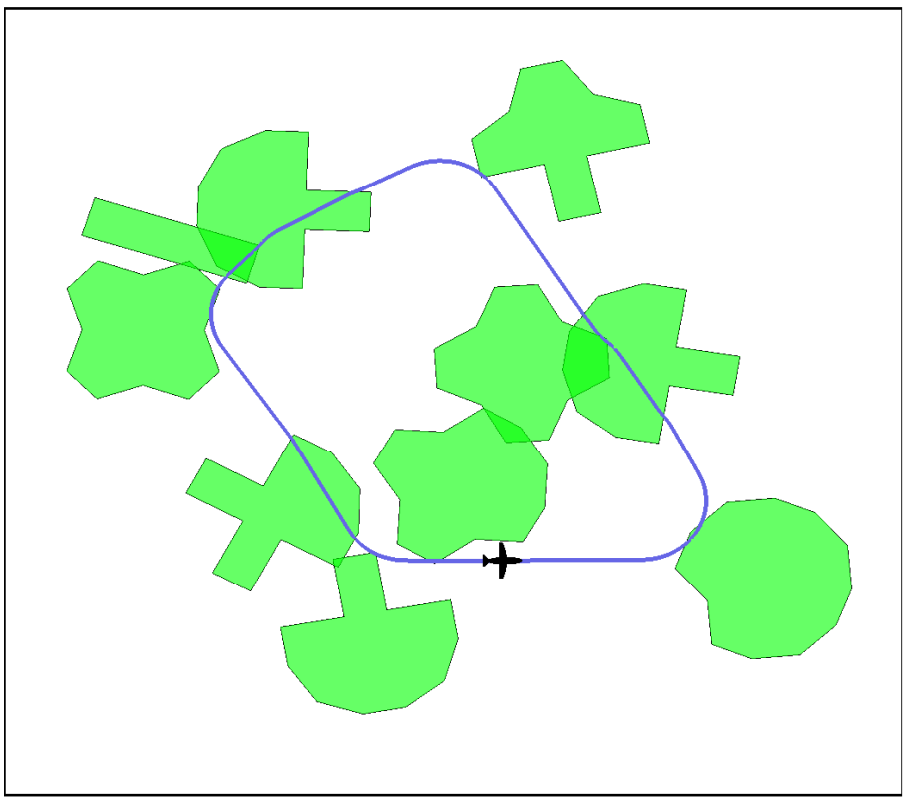




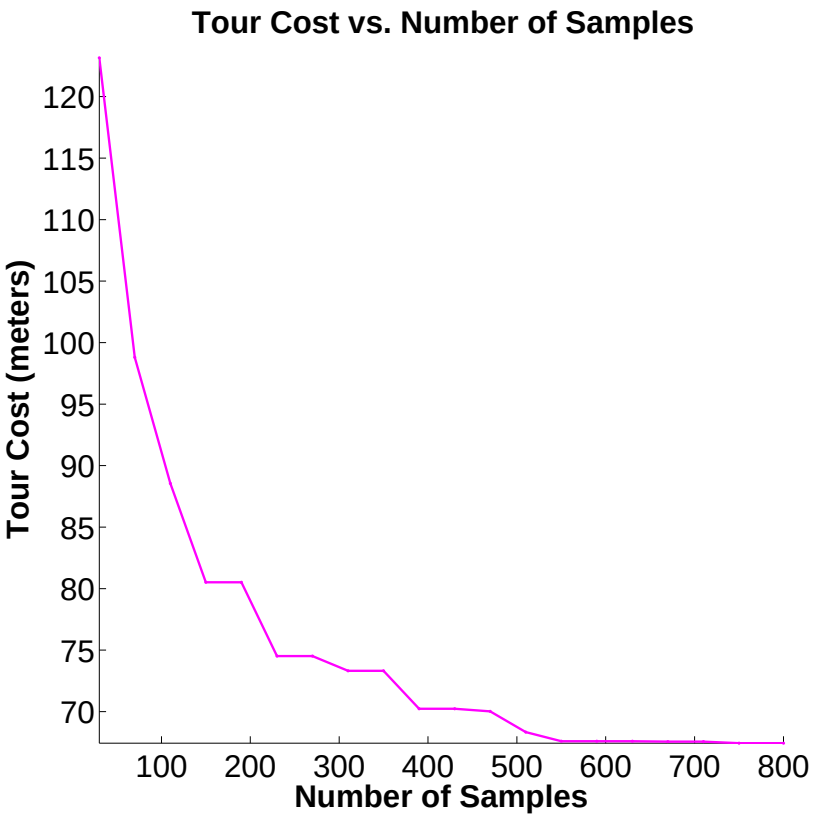
Numerical Study

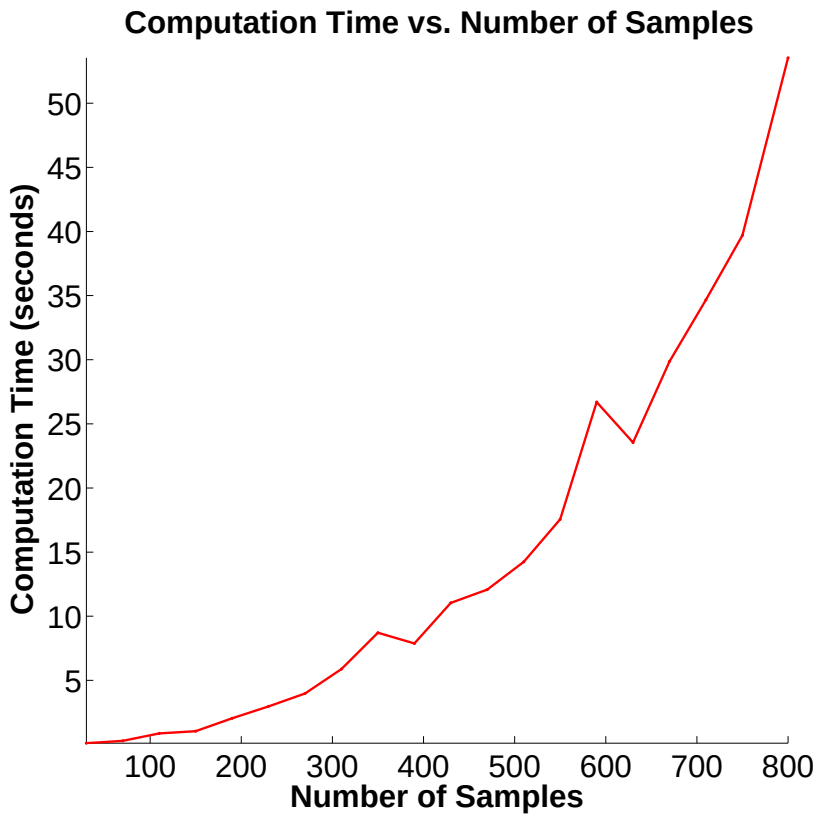
10 Targets, ~ 50 sec. in C++ on 2.33 GHz i686



Numerical Study

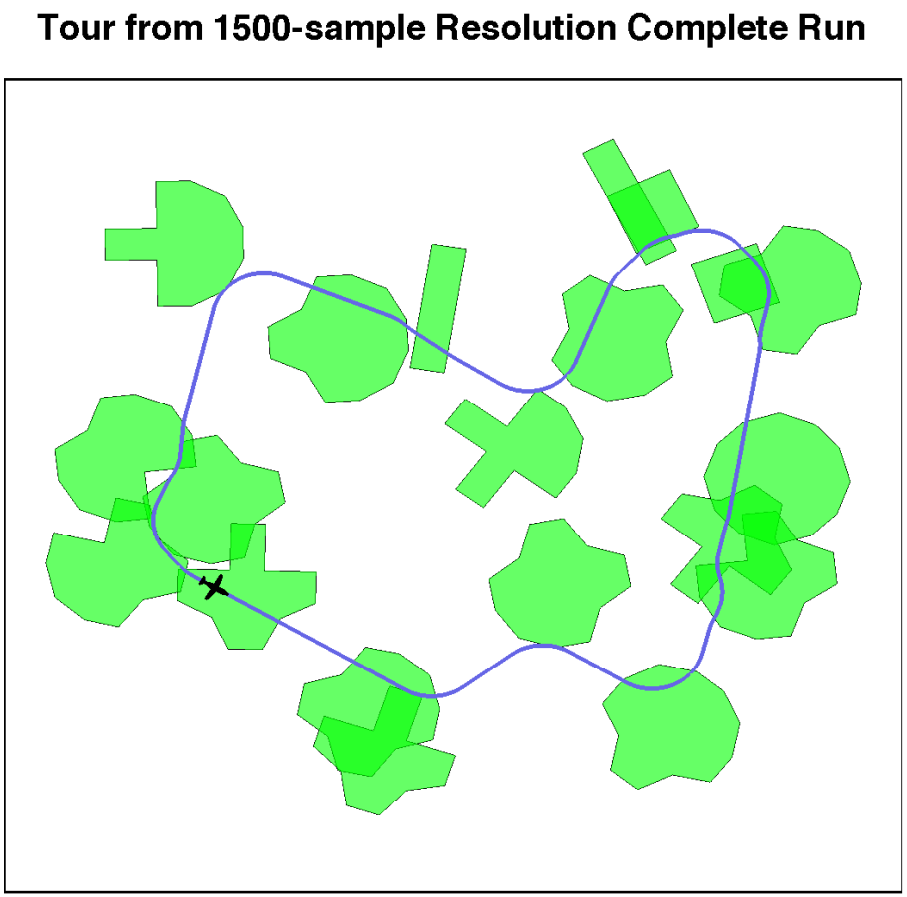
10 Targets, ~ 50 sec. in C++ on 2.33 GHz i686





Numerical Study

20 Targets, ~ 500 sec. in C++ on 2.33 GHz i686



Numerical Study

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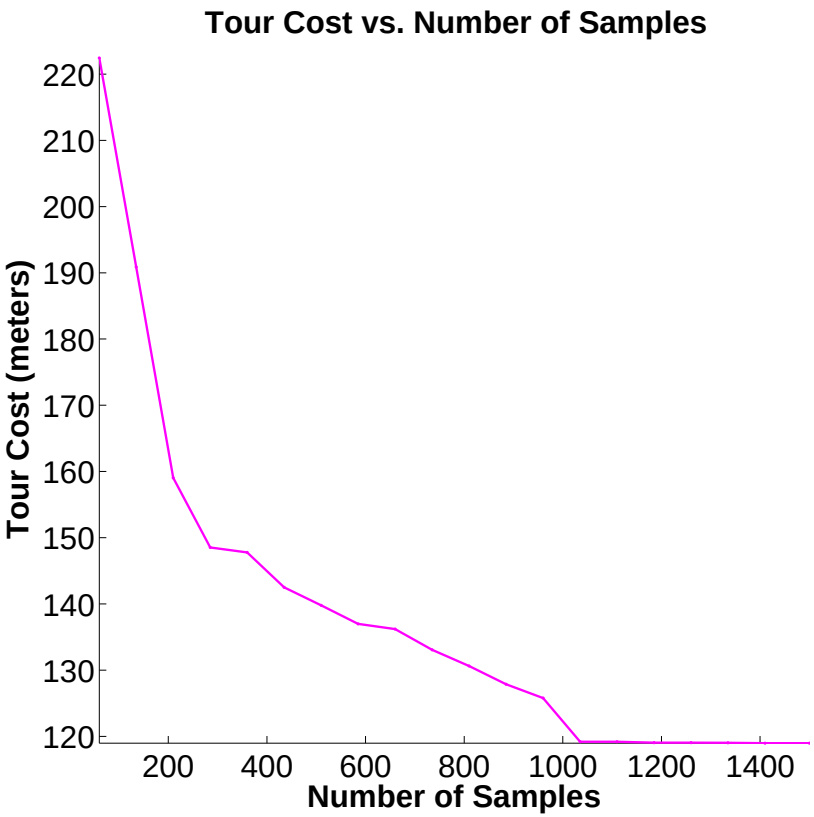
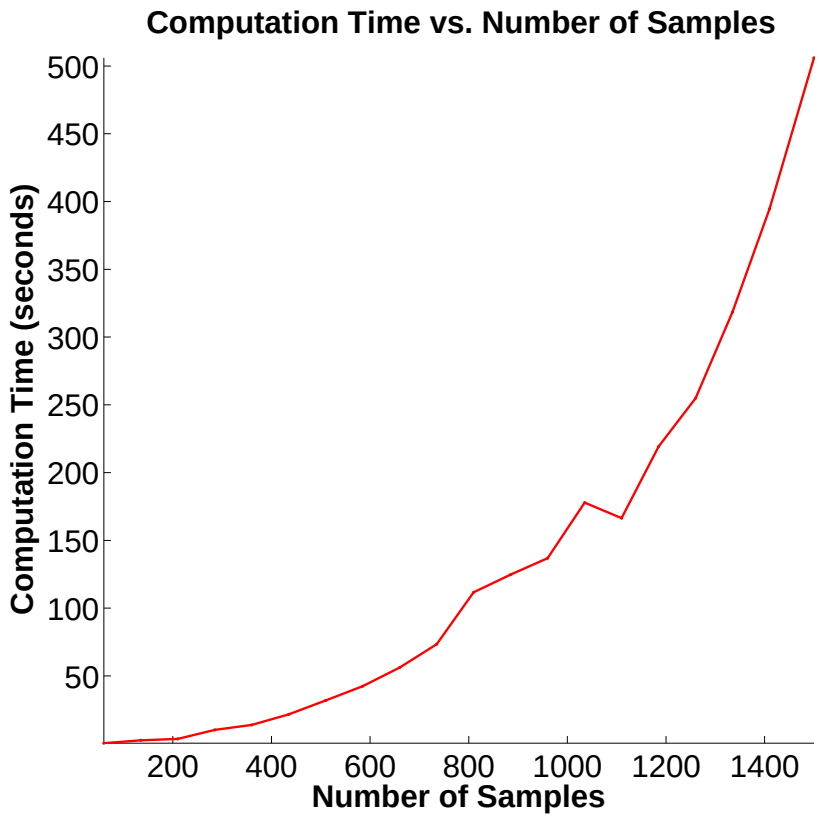




Statistics from Computed Examples

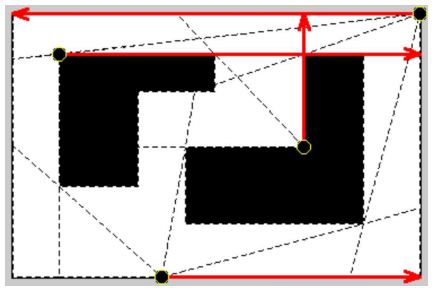
C++, 2.33 GHz i686

Targets	Samples	**Convergence Time**	Tour Length
5	400	**8.05 s**	37.64 m
10	800	**53.54 s**	67.44 m
20	1500	**506.07 s**	118.99 m

Conclusion

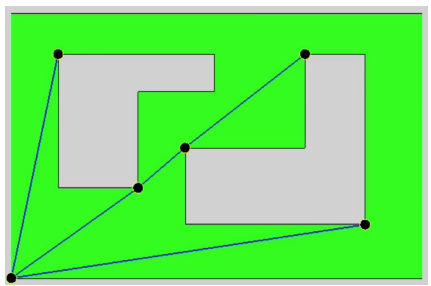
Part I: Searchlight/Camera Scheduling

- Combinatorial roadmap method
- Linear time placement + scheduling



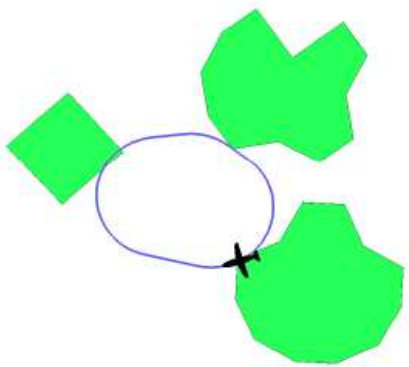
Part II: Multi-Agent Deployment

- Distributed algorithm for environments with holes



Part III: UAV Path Planning

- PVDTSP Formulation
- Sampling-based roadmap method



Visibility-Based Pursuit-Evasion

- Searchlight Scheduling time complexity
- Variations on Searchlight Scheduling
- Minimum time coordinated search with mobile guards

Visibility Coverage

- Practical implementation of deployment
- 3D and dynamic environments
- Optimizing different performance measures

UAV Reconnaissance Path Planning

- Study rate of convergence and parameter sensitivity
- Constant factor approximations
- Multiple vehicles

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