Robust Probabilistic Conflict Prediction for Sense and Avoid

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***Abstract*— Safe integration of Unmanned Aircraft Systems (UAS) into the National Airspace (NAS) will require the development and fielding of a sense-and avoid (SAA) capability to augment the traditional "see-and-avoid" regulations of manned aircraft. In this paper, we focus on the problem of correctly predicting an intruder trajectory. Approaches to intruder prediction are typically grouped into three categories: (i) nominal (deterministic), (ii) worst-case, and (iii) probabilistic. Most prediction algorithms envisioned for SAA fall within the probabilistic category. The benefit of a probabilistic approach is that it provides a mechanism to represent unknown variations in the intruder state at future times while also avoiding the overly conservative assumptions inherent in worst-case prediction. The downside of a probabilistic prediction is that it necessitates the construction of a stochastic model that is both useful for computation and accurately represents the "true" uncertainty in intruder predictions. Markovian structure and time-discretization are very common simplifying assumptions made to satisfy the first goal. Data-driven model tuning is typically used for the latter. However, a model is never exact, and a large quantity of data may be needed to guarantee the approximation accuracy is sufficient.**

The primary contribution of this paper is to present a fourth option for intruder prediction that we refer to as "robust probabilistic prediction." It is meant to address the risk of model mismatch associated with traditional probabilistic predictions. Conceptually, the idea is to specify only those features of the stochastic model that can be justified by data or expert judgment, leaving a full stochastic model underspecified. Typically, one needs a full stochastic model to "turn the crank" on risk calculations (e.g., probability of Near Mid-Air Collision). However, in robust probabilistic predictions, risk is defined as the worst-case risk over a space of stochastic models. This relaxes the need for ensuring that all elements of the model are correct. We show that a computationally efficient semi-definite program (SDP) can be used for performing the optimization over the space of stochastic models. Such an approach greatly reduces the risk of model mismatch as well as reducing the data burden required for model validation.

I. INTRODUCTION

There is a significant need for developing a sense-and-avoid (SAA) capability to allow Unmanned Aircraft Systems (UAS) to safely integrate in the U.S. National Airspace. It is envisioned that automated algorithms and decision aids for detection, intruder prediction, and maneuver planning will fill the gap left by direct visual perception, and this has led to discussions within the UAS and broader aviation communities about the best methods for quantifying the various requirements, performance metrics, and uncertainties needed to drive hardware and algorithm development. The complexity and scale of the problem will necessitate both the development of new technology for "solving" the SAA problem as well as new methods for verifying that developed solutions actually achieve the expected performance when deployed.

Here we investigate the portion of the SAA problem that is concerned with modeling intruders. While the contribution is algorithmic, in so far as we develop methods for quantifying conflict risk, we have an eye toward verification and validation challenges. In this sense, we explore possible algorithmic choices that can decrease the eventual cost of fielding an SAA system.

A. Intruder Prediction Methodologies

Intruder prediction lies at the heart of sense-and-avoid. Indeed, once a potential intruder is detected, the primary function of an SAA system is to predict which action is needed to avoid a future intruder conflict. As a consequence, performing the prediction correctly is central to the success of the overall system. Typically, three prediction methodologies are considered.

- *Nominal (Deterministic) Prediction.* A single trajectory is used to predict the future trajectory of an intruder. Legacy systems such as the Traffic Alert and Collision Avoidance System (TCAS) [1] use a dead-reckoning nominal prediction. Such an approach is conceptually simple and facilitates efficient conflict prediction, but is unable to represent uncertainty in future intruder trajectories. This could introduce significant safety risks in an SAA system, particularly in the presence of maneuvering intruders (e.g., Visual Flight Rules (VFR) traffic, small non-cooperative UAS, etc.).
- *Worst-case Prediction.* A single trajectory is used to predict the future states of an intruder, but the trajectory is optimized to create a conflict. Only the dynamic constraints of the intruder will bound its ability to cause a conflict. While this approach provides the most rigorous safety guarantees, it is impractically conservative for a high density traffic airspace. In addition, the reachability computations required for a worst-case analysis can be computationally expensive.
- *Probabilistic Prediction.* Rather than considering a single trajectory, the probabilistic approach considers a set of possible intruder trajectories. Subsets of trajectories within this set are weighted according to some probability measure that is induced by a particular stochastic model. This prediction methodology natural lends itself to quantifying risk in terms of the probability of future conflict as well as quantifying SAA performance in

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the classical decision theoretic terms of probability of detection and probability of false alarm.

Out of the three prediction methodologies, modern algorithms for SAA tend to focus on probabilistic prediction. Probabilistic prediction seems to offer an acceptable middle-ground between the overly optimistic nominal prediction and an overly conservative worst-case prediction. Further, it provides a mechanism for tailoring SAA system behavior to achieve a particular safety threshold measured, for instance, by probability of Near Mid-Air Collision (NMAC). However, these benefits are only realized if an appropriate stochastic model for trajectory prediction can be developed. Much of the variation in SAA algorithms stems from different answers to this modeling question.

B. Modeling Intruders

Typically, a stochastic model used for intruder prediction must satisfy two needs: (i) accurately represent the uncertainty in an intruder trajectory and (ii) enable the formulation and computation of required risk calculations. Many air-traffic control predictions use Gaussian diffusion models [2]. Data-driven approaches such as the MIT Lincoln Labs Uncorrelated Conflict Model [3] use collected data sets to train a Bayesian net that defines a dynamic Markov model. An approach involving the authors has suggested the use of stochastic maneuver-based models [4] to capture the dynamics the target. However, all such approaches carry the risk that the developed model will not accurately represent the uncertainty in intruder behavior at the time of conflict calculation.

Statistical analysis can help validate a particular stochastic model and mitigate the risk of model mismatch. However, the amount of data required to validate the accuracy of a stochastic process with confidence is significant – requiring validation of both the uncertainty in state at a particular time and the correlations across time. The impact of this need for data is two-fold. First, the intruder models may need to be continually re-calibrated as the statistics of air traffic changes over time – particularly as UAS are fully integrated into the NAS. Indeed, this is a need that the proposed successor to TCAS, Airborne Collision Avoidance System X (ACAS X) [5], anticipates by allowing system updates with new intruder statistics. The second impact is that it is can be difficult to incorporate scenario specific information such as intruder type, known holding patterns, terminal approach routes, etc. into a stochastic model because sufficient data would be needed for each of the multitude of possible combinations.

In this paper, we propose "robust probabilistic prediction" as a principled alternative to traditional probabilistic prediction that simultaneously reduces the risks of model mismatch and data validation requirements. Rather than specify a *particular* stochastic model, we propose to specify a *space* of possible stochastic models based on a small number of aggregate statistics. A worst-case risk calculation is then performed by optimizing over the space of stochastic models. Such an approach greatly reduces the chance for model mismatch by reducing the number of model parameters. In addition, it provides a method for inserting expert judgment to perform probabilistic risk calculations when empirical data is limited or lacking. The methods for solving this problem utilize semi-definite programming (SDP) [6] for which fast interior-point solution methods are available. The size of the proposed SDP is small, involving matrices on the order of 10×10. Recent work developing interior-point solvers for embedded systems [7] suggest that the proposed solution methods could be made feasible for real-time operations. In addition, this analysis technique could greatly reduce the field testing requirements by quantifying the degree to which specific stochastic modeling assumptions would actually influence the behavior of the SAA system.

The remainder of this paper is laid out as follows. In Section II we show that an intruder stochastic model used for conflict risk prediction can be reduced to a finite-dimensional random variable. This is the first step in reducing a stochastic process to a more manageable object. In Section III, we develop the semi-definite program used for performing the robust conflict prediction. Section IV provides an illustrative example for proposed approach, and we provide concluding remarks in Section V.

II. Stochastic Processes for Conflict Prediction

A continuous-time, stochastic process is an infinite-dimensional object. Even for the limited subset of Markov processes, it can be very difficult to put useful and meaningful bounds on the space. However, in SAA applications, the ultimate use of a stochastic process is to provide methods for quantifying future conflict risk, and for this particular application a much simpler, finite dimensional object is sufficient. It is worth noting that our approach avoids discretizing the time dimension. While such an approach would produce a strictly, finite-dimensional random variable, the number of needed dimensions would explode if a very fine-discretization were needed to capture the fast time scales of a conflict event. The proposed approach maintains a continuous time dimension to avoid these concerns.

In the following, we will assume that vector $X(t)$ represents the intruder trajectory over a time horizon $t \in [0, T]$. Without loss of generality, we assume that the time variable t is appended to the state vector $X(t)$ and that $X(t)$ has been normalized with respect to the ownship trajectory so that the conflict region C is a fixed subset of the state space. In the case of NMAC, this region C would be the protection volume of the ownship. Other conflict regions are possible as well. For instance, a region C that is useful for the Self-Separation (SS)S function of sense-and-avoid [8] may define the state in which the predicted closing rate is less than some threshold because this would activate the Collision Avoidance (CA) function. We assume that conflict occurs if $X(t)$ penetrates the region C at any point during a time horizon $[0, T]$ and we are primarily interested in the probability that this occurs. The following claim establishes that the stochastic process $X(t)$ can be equivalently represented by a finite-dimensional random vector Y.

Claim 1: Let $X(t) \in \mathcal{R}^n$ be an n-dimensional, real-valued, continuous stochastic process defined over a time horizon $t \in [0, T]$, and $C \subset \mathcal{R}^n$ be a closed region defining "conflict" states. Let P_c denote the probability of conflict at some point over a time horizon.

$$P_c = P\left[\bigcup_{t\in[0,T]} \{X(t) \in C\}\right]$$

Then there exists an n-dimensional random variable Y such that

$$P[Y \in C] = P_c.$$

Proof: We prove this claim by explicit construction of Y. Let Ω define the sample space for the stochastic process $X(t)$ and the random variable Y. Let $X(t;\omega)$ define the sample path associated with $\omega \in \Omega$. Let $T_c(\omega)$ denote the possibly empty subset of conflict times

$$T_c(\omega) \triangleq \{t \mid X(t,\omega) \in C,\, 0 \le t \le T\}.$$

Note that T_c is a closed set because X is continuous and C is closed. We will define the time random variable τ^* as

$$\tau^*(\omega) \triangleq \begin{cases} 0 & T_c(\omega) = \emptyset \\ \inf T_c(\omega) & \text{otherwise} \end{cases} \tag{1}$$

Define Y as

$$Y(\omega) \triangleq X\left(\tau^*(\omega);\omega\right)$$

Assume there exists a t such that $X(t;w) \in C$, then $T_c(\omega)$ is non-empty. Further, $\inf T_c(\omega) \in T_c$ and thus $Y(\omega) \in C$. This implies that $P[Y \in C] \ge P_c$. On the other hand, if there does not exist t such that $X(t;\omega) \in C$, then $X(0;\omega) \notin C$ and thus $Y(\omega) \notin C$ implying $P_c \ge P[Y \in C]$.

■

III. Robust Probabilistic Conflict Risk

The random variable Y constructed in Claim 1 provides a vehicle for bounding the space of stochastic processes by bounding the probability laws for Y. We do this by bounding the mean and covariance for Y, and then leveraging generalized Chebyshev bounding techniques [9]. To understand how such bounds are constructed, we return to the the stochastic process $X(t)$ and decompose it into a "deterministic" part and a "stochastic" part. Let

$$X(t;\omega) = \bar{X}(t) + \Delta(t;\omega).$$

In this decomposition, $\bar{X}(t)$ can be the mean trajectory $\mathbb{E}X(t)$, but it is not strictly required for what follows. $\Delta(t;\omega)$ is variation from this nominal path. Note that this decomposition induces a similar decomposition for Y

$$Y(\omega) = \bar{X}(\tau^*(\omega)) + \Delta(\tau^*(\omega),\omega) \tag{2}$$

Thus Y can be thought of as the sum of two random vectors $\bar{X}$ and Δ. Note that while, $\bar{X}(t)$ is the deterministic component of $X(t)$, the vector $\bar{X}(\tau^*(\omega))$ is a stochastic quantity due to the stochastic nature of the evaluation time. Our approach to bounding the moments for Y is to bound the moments of $\bar{X}(\tau^*(\omega))$ and $\Delta(\tau^*(\omega);\omega)$.

A. Semi-Definite Programming Bounds

For the quantity $\bar{X}$, we know the support for the distribution. It is exactly the bounded subspace $\{X(t) \mid t \in [0,T]\}$, and this provides a straight-forward way to bound the statistics. The bounds for Δ are related to the variations of the stochastic process. We present a method for defining convex bounds on Δ from moment trajectories of $X(t)$ in Section III-B. With bounds on $\bar{X}$ and Δ, we can compute a worst-case bound on the conflict probability. If the conflict region C is defined by a quadratic form, the optimization can be performed via semi-definite programming.

Claim 2: Let $\bar{X}(t)$ be the nominal path for an n-dimensional stochastic process $X(t)$ given by

$$\bar{X}(t) = \sum_{i=0}^{m} P_i b_{i,m}(t/T) \tag{3}$$

where $b_{i,m}$ are Bernstein polynomials of degree m and P_i are control point vectors. Let $H = \{(\eta_j, a_j)\}$ be the set of hyperplanes defining facets of $conv(\{P_i\})$ expressed as tuples of inward facing normal vectors and offsets.

Let $\bar{X}, \Delta$ be random variables defined as in (2). Assume Δ is bounded by the a set of constants constraints $\{(c_{ij}^-, c_{ij}^+)\}$ for $(i,j) \in [1...n] \times [1...n]$ according to

$$c_{ij}^- \le \mathbb{E}\Delta_{ij}^2 \le c_{ij}^+$$

Let the conflict region C be defined by a quadratic constraint

$$C = \left\{x \in \mathcal{R}^n \mid \text{trace}(Axx^T) + 2b^T x + c \le 0\right\} \tag{4}$$

Then

$$P_c \le \lambda$$

where λ is given by the outcome of the following semi-definite program.

$$\begin{aligned}
\min \quad & 1 - \lambda \\
\text{s.t.} \quad & \begin{bmatrix} Z & z \\ z^T & \lambda \end{bmatrix} \succeq 0 \\
& \begin{bmatrix} Y_{11} & Y_{12} & y_1 \\ Y_{21} & Y_{22} & y_2 \\ y_1^T & y_2^T & 1 \end{bmatrix} \succeq 0 \\
& \begin{bmatrix} Z & z \\ z^T & \lambda \end{bmatrix} \preceq \begin{bmatrix} Y_{11} + Y_{12} + Y_{21} + Y_{22} & y_1 + y_2 \\ y_1^T + y_2^T & 1 \end{bmatrix} \\
& \text{trace}(AZ) + 2b^T z + c\lambda \le 0 \\
& \textit{Bounds for } \bar{X}\text{:} \qquad (i, j = 1, ...|H|) \\
& \eta_j^T y_1 \ge a_j \\
& \text{trace}(\eta_j \eta_i^T Y_{11}) - (a_i\eta_j + a_j\eta_i)^T y_1 + a_i a_j \ge 0 \\
& \textit{Bounds for } \Delta\text{:} \qquad (i, j = 1, ..., n) \\
& c_{ij}^- \le [Y_{22}]_{ij} \le c_{ij}^+
\end{aligned}$$

Proof: Using Claim 1, there exists a Y such that $P_c = P[Y \in C]$. An upper bound for P_c follows from constructing a lower bound for $P[Y \notin C]$. This lower bound is computed by relaxing the SDP formulation for generalized Chebyshev bounds of [9] by replacing known moments on Y with bounds on moments. These unknown moments are represented by the auxiliary variables Y and y with Y_{11}, y_1 representing the moments for $\bar{X}$ and Y_{22}, y_2 the moments for Δ. The support for $\{X(t) \mid t \in [0,T]\}$ is contained within the convex hull of $\{P_i\}$ using a convex hull property of Bernstein polynomials. Thus the mean of $\bar{X}$ must fall within this set. Further, these bounds limit the possible second moment for $\bar{X}$ along with the normal vectors of this convex hull. The bounds on Y_{22} follow directly from assumed bounds on $E[\Delta_{ij}^2]$. ■

B. Bounds from Moment Trajectories

The purpose of robust probabilistic predictions for SAA is to make it easier to faithfully represent uncertainties in intruder motion by decreasing the number of modeling decisions and decreasing the data burden required to validate those decisions. It is in this spirit, that we explore partial representations of intruder uncertainty that are "natural" or otherwise amenable to input from experts and/or empirical data analysis. A very common method for representing uncertainty in a stochastic processes is through moment trajectories (e.g., $\mathbb{E}X^k(t)$). Such trajectories are insufficient for completely specifying the stochastic model but they are intuitively appealing (e.g., nominal trajectory, covariance growth over time, etc.) and straight-forward to estimate from empirical data because correlation across time is not needed. As such, we take moment trajectories as our assumed incomplete description for the stochastic process. In this section we will show how bounds for the moments on the variable $\Delta(\tau^*(\omega);\omega)$ can be obtained using moment trajectories of the form $\{\mathbb{E}[X_i^{k_1}(t)\dot{X}_i^{k_2}(t)]\}$ for $k_1, k_2 \in \{0,1,2\}$. Explicit quantification of correlation between time periods is not needed.

We will begin with a pair of simple lemmas that are useful for constructing bounds.

Lemma 1: Let $Z(t)$ be a differentiable, time-varying function. Then

$$\sup_{t\in[0,T]} Z(t) \le \frac{Z(0)+Z(T)}{2} + \frac{T^{1/2}}{2}\left(\int_0^T \dot{Z}^2(s)\,ds\right)^{1/2}. \tag{5}$$

Proof: This result follows from a classical result in Parzen [10]. Note that

$$Z(t) = Z(0) + \int_0^t \dot{Z}(s)\,ds$$

and similarly

$$Z(t) = Z(T) - \int_t^T \dot{Z}(s)\,ds$$

so that

$$Z(t) = \frac{1}{2}\left(Z(0)+Z(T)\right) + \frac{1}{2}\int_0^T \operatorname{sgn}(t-s)\dot{Z}(s)\,ds$$

From here we have

$$\begin{aligned}\int_0^T \operatorname{sgn}(t-s)\dot{Z}(s)\,ds &\le \int_0^T |\dot{Z}(s)|\,ds \\ &\le T^{1/2}\left(\int_0^T \dot{Z}^2(s)\,ds\right)^{1/2}\end{aligned}$$

The right hand side is not dependent on t, so taking supremum over t yields the result. ■

Corollary 1: This analysis can be easily extended to provide lower bounds for the Z as well.

$$\inf_{s\in[0,T]} Z(s) \ge \frac{Z(0)+Z(T)}{2} - \frac{T^{1/2}}{2}\left(\int_0^T \dot{Z}^2(s)\,ds\right)^{1/2}$$

A time-dependent bound on $Z(t)$ also holds.

Lemma 2: Let $\|\dot{Z}\|_\infty = \sup_{s\in[0,T]} |\dot{Z}(s)|$. Then

$$Z(0) - t\|\dot{Z}\|_\infty \le Z(t) \le Z(0) + t\|\dot{Z}\|_\infty \tag{6}$$

Now we are in a position to obtain bounds for Δ.

Claim 3: Let

$$D_{ij}(t;\omega) \triangleq (X_i(t;\omega) - \bar{X}_i(t))(X_j(t;\omega) - \bar{X}_j(t)).$$

Then the following bounds hold:

$$\begin{aligned}\mathbb{E}\left[\Delta_i\Delta_j\right] &\lesseqgtr \frac{1}{2}\left(\mathbb{E}D_{ij}(0) + \mathbb{E}D_{ij}(T)\right) \\ &\pm \frac{T^{1/2}}{2}\left(\int_0^T \mathbb{E}\left[\dot{D}_{ij}(s)^2\right]\,ds\right)^{1/2} \quad (7)\\ \mathbb{E}\left[\Delta_i\Delta_j\right] &\lesseqgtr \mathbb{E}[D_i(0)D_j(0)] \pm \mathbb{E}\left[\tau^*\|\dot{Z}\|_\infty\right] \quad (8)\\ \mathbb{E}\left[\Delta_i\Delta_j\right] &\lesseqgtr \mathbb{E}[D_i(T)D_j(T)] \\ &\mp T\,\mathbb{E}\left[\|\dot{Z}\|_\infty\right] \pm \mathbb{E}\left[\tau^*\|\dot{Z}\|_\infty\right]. \quad (9)\end{aligned}$$

Proof: Let $\Delta_{ij}(\omega) = \Delta_i(\tau^*(\omega);\omega)\Delta_j(\tau^*(\omega);\omega)$, and note that

$$\inf_{s\in[0,T]} D_{ij}(s;\omega) \le \Delta_{ij}(\omega) \le \sup_{s\in[0,T]} D_{ij}(s;\omega).$$

The first bound (7) follows directly by taking expectations (with respect to Y) and applying Jensen's inequality to the result from Lemma 1. The second and third bounds (8) and (9) follow by taking expectations of Lemma 2. ■

The moment bounds for $\Delta_i\Delta_j$ expressed in (7) provide the constant bounds for the Δ variables shown in the SDP formulation from Claim 2. Intuitively, we would like to have bounds for Δ that are coupled to the statistics of the conflict time τ^*. For instance, if the uncertainty in state $X(t)$ (measured by deviation from $\bar{X}(t)$) increases across the time horizon, this should be reflected in the moment bounds for Y. The inequalities (8) and (9) provide such a mechanism. If we assume that $X(t)$ already includes t as a state variable, then utilizing these time-correlated bounds merely requires the augmentation of the SDP to include additional variables $\{\|\dot{D}_{ij}\|_\infty\}$. Moment bounds for these new variables can be provided directly (i.e., a further specification of the stochastic model), or through the same process as was used to obtain the constant bounds for $\Delta_i\Delta_j$ variables. This later step would involve an indirect constraint on the acceleration moments $\mathbb{E}\left[\dot{X}(t)^{k_1}\ddot{X}(t)^{k_2}\right]$ for $k_1, k_2 \in \{0,1,2\}$.

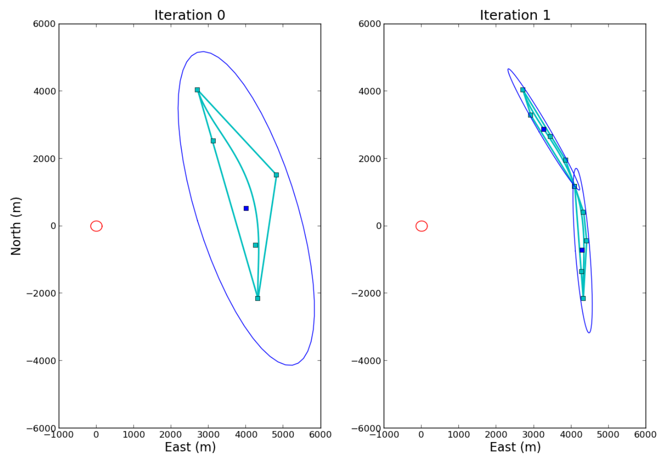

Fig. 1. Time-splitting improves bounds on $\bar{X}$ variables. The worst-case mean and covariance for $\bar{X}$ obtained from the SDP optimization is shown in dark blue. The conflict region C is shown in red. For the initial iteration the mean of $\bar{X}$ lies significantly off the nominal trajectory. A single refinement improves this considerably. The corresponding worst case bounds go from $P_c \leq 0.13$ in iteration 0 to $P_c \leq .006$ in iteration 1.

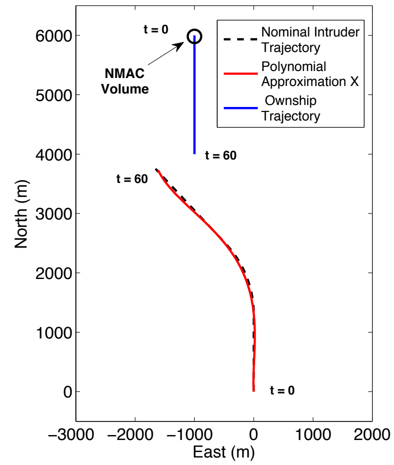

Fig. 2. Geometry for an example encounter. The ownship, heading due South, encounters an intruder whose future trajectory is uncertain, but whose nominal (mean) path includes a change of heading from North to North-West.

C. Iterative Refinement through Time-Splitting

In the SDP formulation from Claim 2, moments for $\bar{X}$ are bounded through a convex hull of the trajectory $\{X(t) \mid t \in [0,T]\}$. If this trajectory differs significantly from straight-line motion, then the developed bounds could be extremely loose. Figure 1 demonstrates this phenomena. However, it is possible to mitigate this effect through a time-splitting process. Conceptually, the idea is to partition the time axis so that $[0,T] = \cup_i \sigma_i$ and a convex hull is constructed to bound $\bar{X}$ in each partition.

No essential change in the approach is needed for the variation, we only required an augmentation of Y. We introduce a partition indicator function

$$\delta_i(t) \triangleq \begin{cases} 1 & t \in \sigma_i \\ 0 & \text{otherwise} \end{cases}$$

and let

$$Y^{(i)}(\omega) \triangleq \delta_i(\tau^*(\omega))Y(\omega)$$

The $\bar{X}$ variables associated with each $Y^{(i)}$ are then bounded by tighter convex hull for the trajectory $\{X(t) \mid t \in \sigma_i\}$. In this formulation, we have

$$P_c = P\left[\bigcup \{Y^{(i)} \in C\}\right] = 1 - P\left[\bigcap_i \{Y^{(i)} \in \bar{C}\}\right] \quad (10)$$

We define new random variable $\hat{Y}$ by stacking the $Y^{(i)}$ variables

$$\hat{Y} \triangleq \begin{bmatrix} Y^{(1)} & \cdots & Y^{(n)} \end{bmatrix}^T$$

and define $C^{(i)}$ to obey

$$\hat{Y} \in C^{(i)} \iff Y^{(i)} \in C.$$

Then we can re-write (10) as

$$P_c = 1 - P\left[\bigcap_i \{\hat{Y} \in \bar{C}^{(i)}\}\right].$$

The SDP formulation from [9] accommodates an intersection of quadratic constraints, allowing the minimization over probability laws for $\hat{Y}$ to optimized as before. Note that each partition increases the number of optimization variables in the SDP *linearly* rather than quadratically as might be expected. This is because the δ_i indicator functions induce a block diagonal structure on $\mathbb{E}[\hat{Y}\hat{Y}^T]$. Though a small number of well-chosen refinements typically suffice, the linear growth in complexity ensures that the partitioning approach does not create a significant computational burden even if the number of partitions is large.

IV. EXAMPLE

In this section, we demonstrate robust probabilistic prediction on an simple two-dimensional example involving an uncertain intruder. We consider a scenario in which the nominal path intruder is flying at 140 knots and executing a coordinated turn left. We consider an ownship that is moving at a notional speed of 80 knots due South and at coincident altitude. We parameterize the staring location, and calculate bounds on the probability that NMAC occurs at some point within a 60 second time horizon. The geometry of the example encounter is shown in Figure 2.

In practice the moment trajectories used to bound the variation around the nominal path would be generated by empirical data analysis and/or expert judgment. Here we simulate this process by sampling from a diffusion process with a "drift" that follows the notional path and computing sample averages. The trajectories $\{\mathbb{E}\left[(X_1(t) - \bar{X}_1(t))^2\right]^{1/2}, \mathbb{E}\left[(X_2(t) - \bar{X}_2(t))^2\right]^{1/2}\}$ produced through this process are show in Figure 3. Using Claim 3, we calculate the constant bounds on Δ_1, Δ_2.

$$\mathbb{E}\Delta_1^2 \leq 2.73e4 \quad \mathbb{E}\Delta_1^2 \leq 1.53e4$$

Using these bounds (and trivial lower bounds) along with the polynomial fit for the nominal trajectory, we can compute

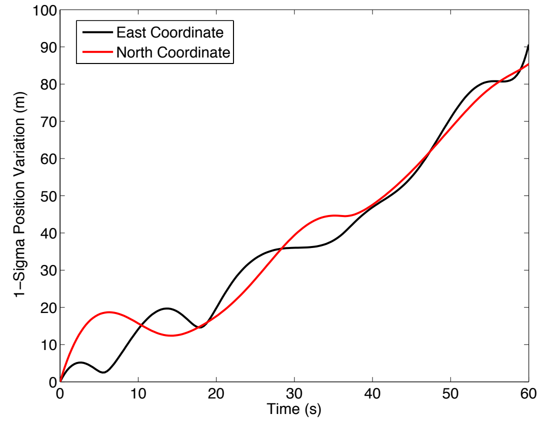

Fig. 3. Moment trajectory for expected variation in position coordinates (e.g., $\mathbb{E}\left[(X_1(t) - \bar{X}_1(t))^2\right]^{1/2}$). Variation in the trajectory is due to the polynomial approximation of the mean trajectory.

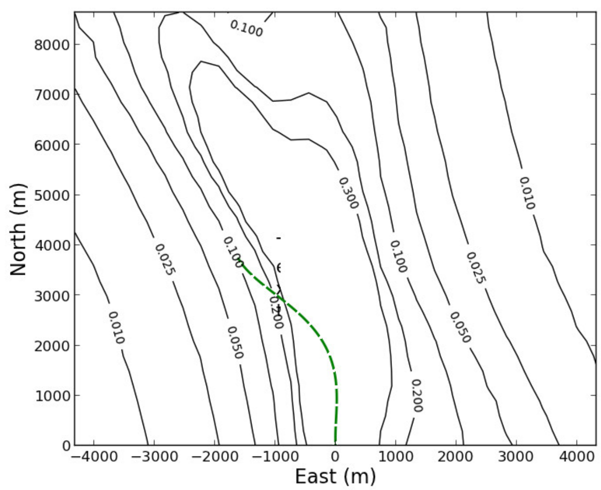

Fig. 4. Contour map for worst-case P_c bounds as a function of the starting location of the ownship. The nominal intruder trajectory is shown by a dashed line.

worst-case bounds for the probability of NMAC across a range of ownship locations. A total of two time-splitting refinements was used for to generate the contours. The results are shown in the contour map in Figure 4.

V. CONCLUSION

In this paper, we have presented an approach to conflict prediction for sense-and-avoid to mitigate the impact of model mismatch for intruder prediction. This was done by relaxing the requirements needed to specify a complete probabilistic model and instead requiring only partial specification of a stochastic process in terms of moments trajectories. An SDP formulation is used to determine worst-case upper bounds on the conflict probability by maximizing over the space of stochastic processes.

This work suggest some interesting extensions for future research. The most obvious would involve an exploration of how tight the developed bounds can be made, and to compare these bounds against those generated by specific intruder prediction models such at Lincoln Labs uncorrelated encounter model [3]. A related analysis task would be to use the same methods for computing lower bounds on P_c. While less useful in an operational system, both upper and lower bounds will aid in understanding the extent to which the details of a stochastic model actually matter for the conflict risk calculations. In addition to this analysis, there are two extensions to the developed approach that that may be particularly useful for researchers developing SAA solutions. The first is to study techniques for avoidance maneuver planning that could leverage the SDP prediction approach. Approaches such as ACAS X use a Markov Decision Process (MDP) formulation that makes strong assumptions about the intruder model (e.g., it's a Markovian stochastic process) and it would be interesting to understand the degree to which these assumptions could be relaxed. A second, perhaps more ambitious direction would pursue the extent to which the robust prediction formulation could be used to relax the "blunder scenario" assumptions inherent in existing stochastic models. Current approaches assume that the intruder does not respond to maneuvers of the ownship (i.e., it "blunders" into the ownship) so that there is no feedback in the intruder trajectory. This is clearly a modeling assumption that is incorrect for some conflict scenarios, particularly for the longer time scale conflict associated with the self separation (SS) function of sense-and-avoid. Extending robust analysis to this modeling question could help provide safety guarantees for a SAA system in a broader range of encounters.

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